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Application of non-linear and temporal analysis in physical and engineering programs

24th Summer School and Conference "Dynamical Systems and Complexity", University of Thessaly, Volos, Greece

Plan of Presentation

- Introduction to time series
  - Linear Methods
  - Non Linear Methods
- Temporal behavior analysis
  - Power spectrum
  - Hurst exponent
  - Average mutual information
- Phase space reconstruction methods
  - Recurrence plots
- Temporal Correlations – Causality
- Time series to Complex networks transformation
- Applications in experimental and field measurements
  - Turbulent heated jet
  - River water level
  - Rainfall
  - Environmental data

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  - rainfall

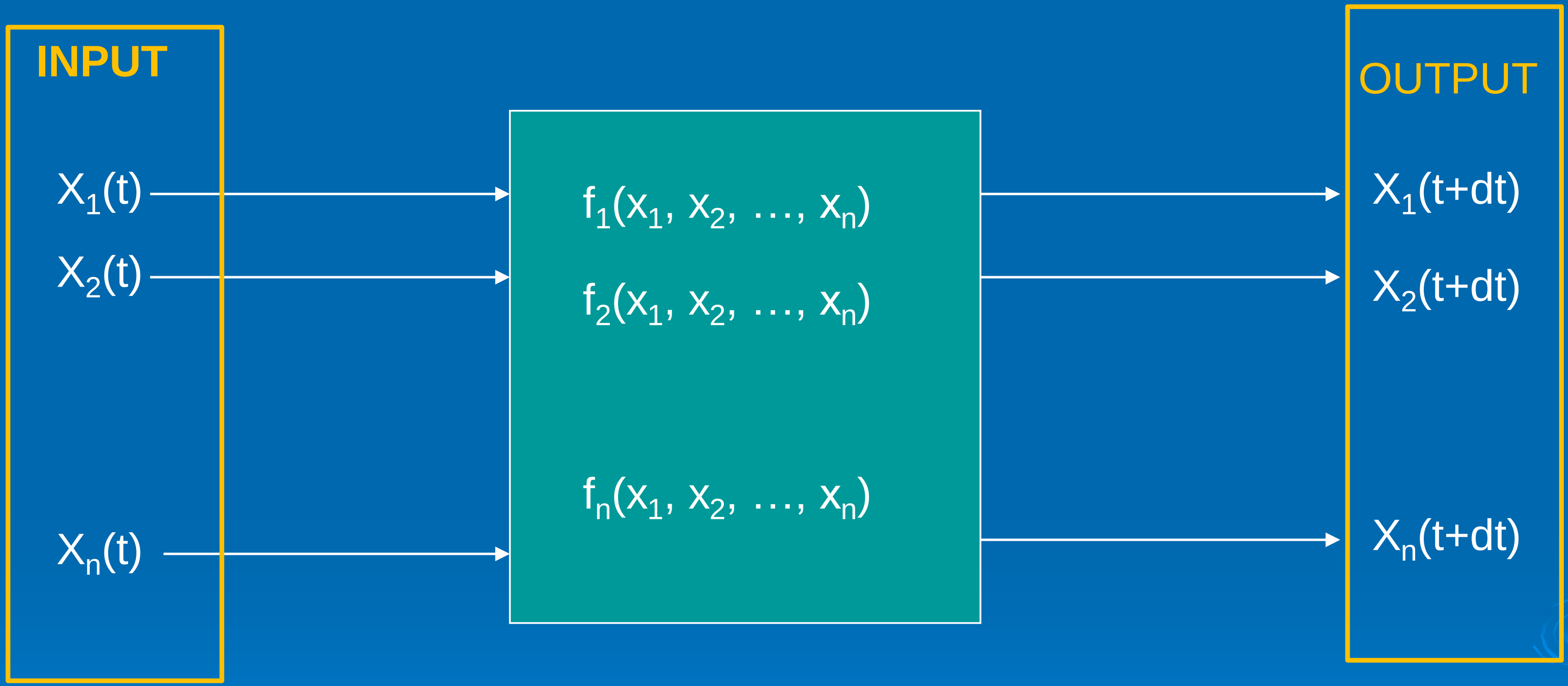
Time series

- An observed dynamical system (simulated / experimental / field measurements) results in observables varying in time → time series

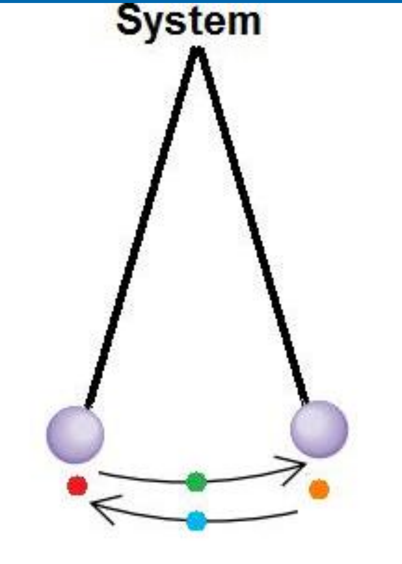
- In engineering and environmental systems we are aware of system monitoring

- Do time series contain information about the underlying system dynamics that can be useful?

Dynamical System

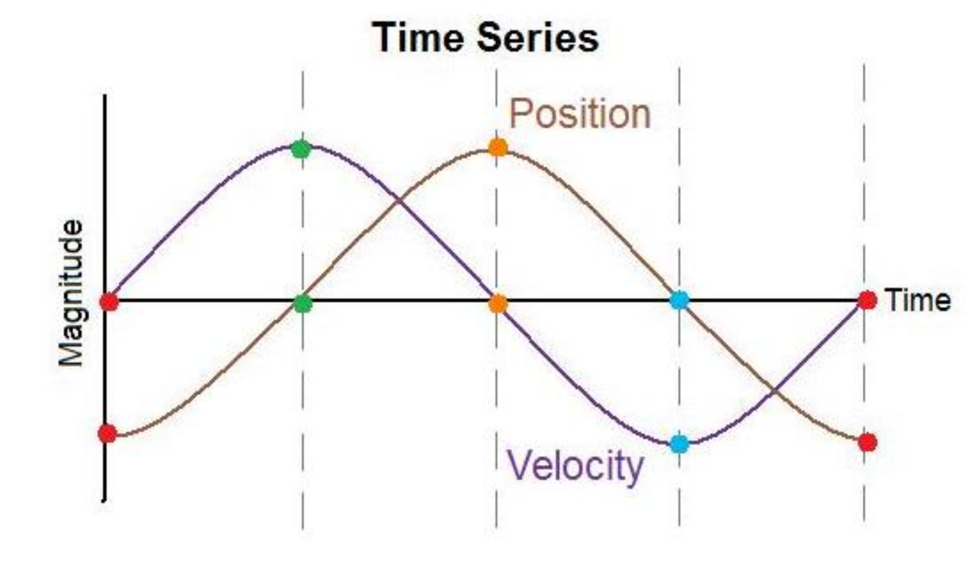


Example (1) - Pendulum
Known Dynamical System → Time Series

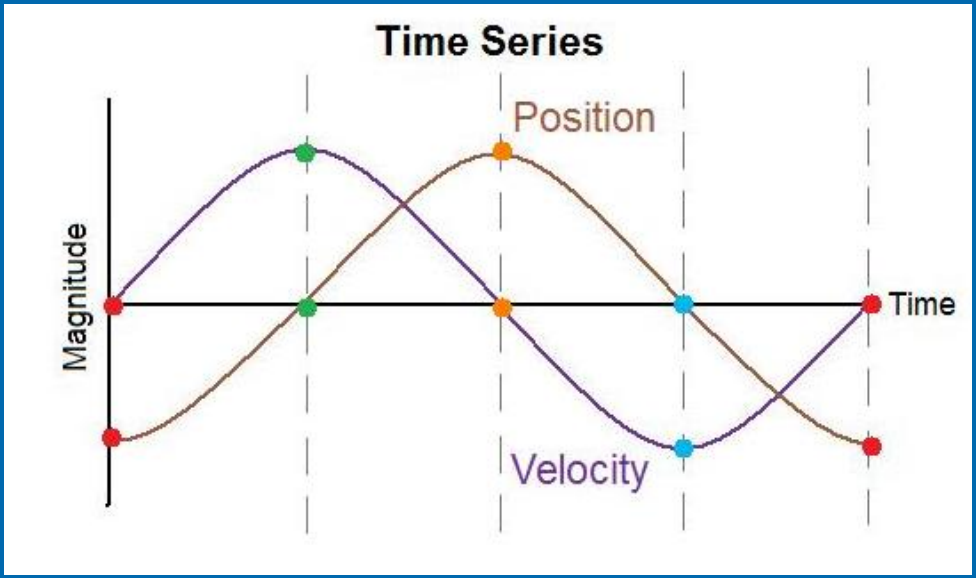


$$\dot{\theta}(t) = \omega(t)$$

$$\dot{\omega}(t) = -\frac{g}{L}\sin\theta(t)$$



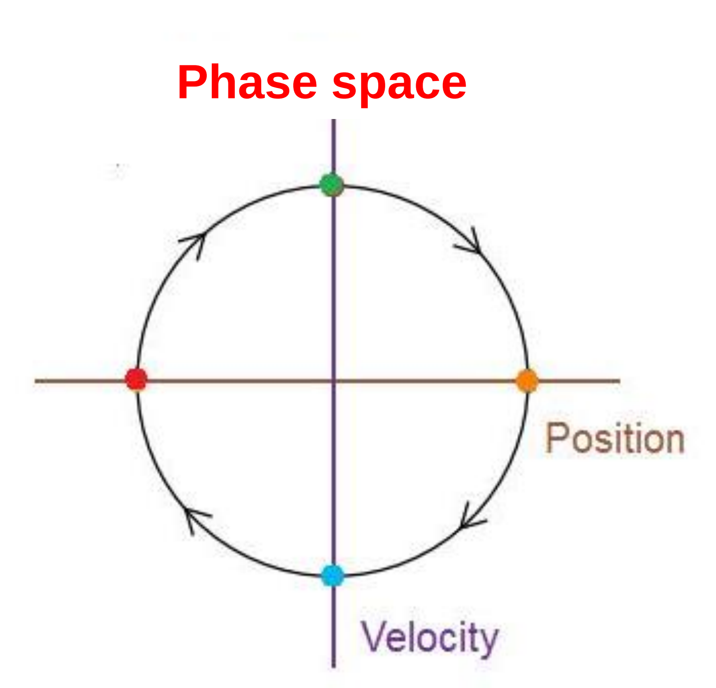
Example (1) - Pendulum from time series to phase space



Plot for every time t all points(θ, ω)

From time

To Geometry



Example (2)
Known dynamical system ⇨ time series

- Lorentz system (chaotic)

$$\frac{dx_1}{dt} = \sigma(x_2 - x_1),$$
$$\frac{dx_2}{dt} = rx_1 - x_2 - x_1 x_3,$$
$$\frac{dx_3}{dt} = x_1 x_2 - b x_3,$$

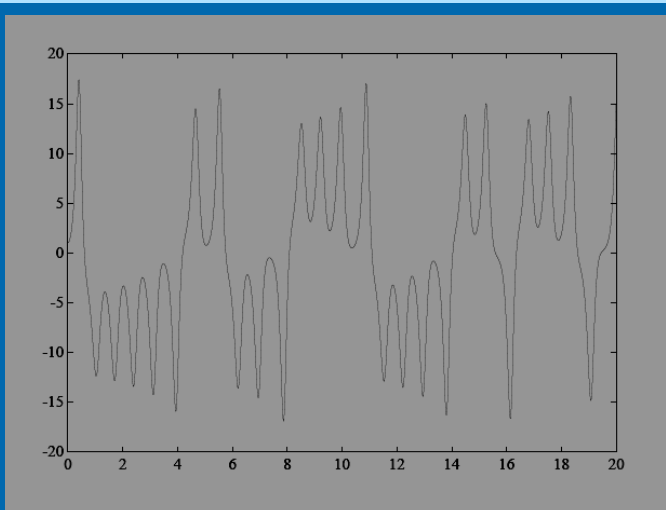
$\sigma = 10$, $b = \frac{8}{3}$, and $r = 28$.

Initial conditions

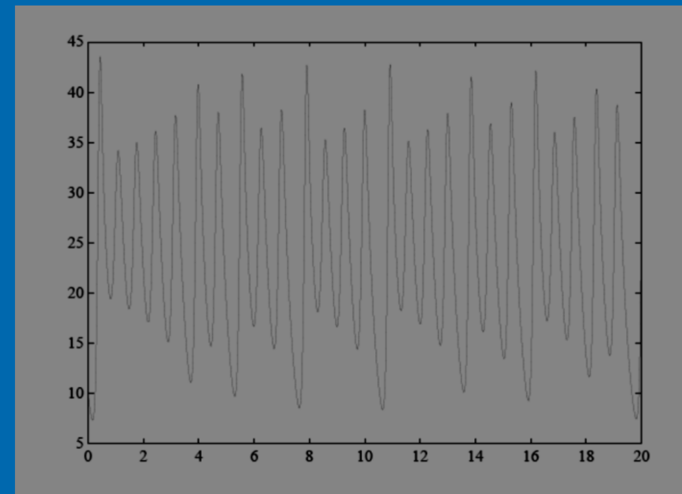
$$\mathbf{x}_0 = \begin{bmatrix} 1 \\ 1 \\ 10 \end{bmatrix}$$

Example Lorentz (cont'nd)

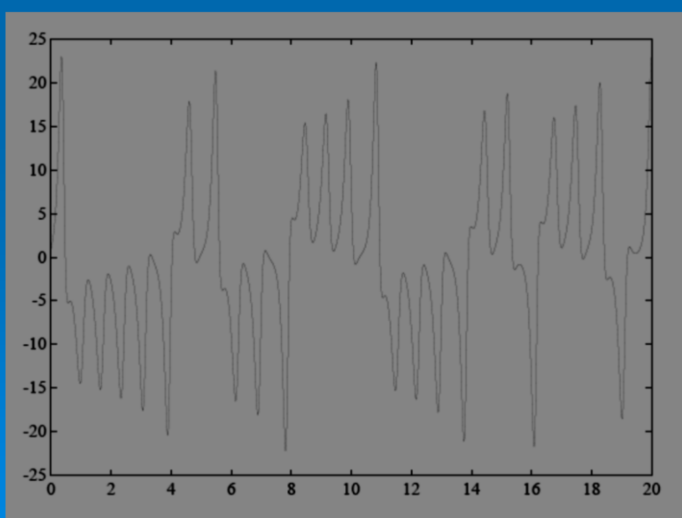
$x_1(t)$



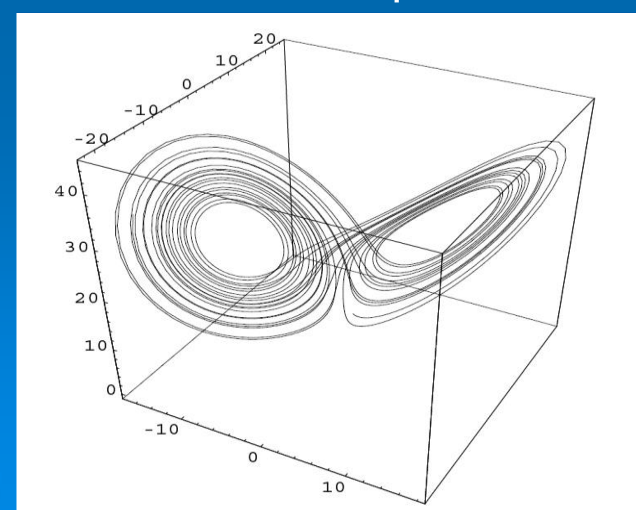
$x_3(t)$



$x_2(t)$

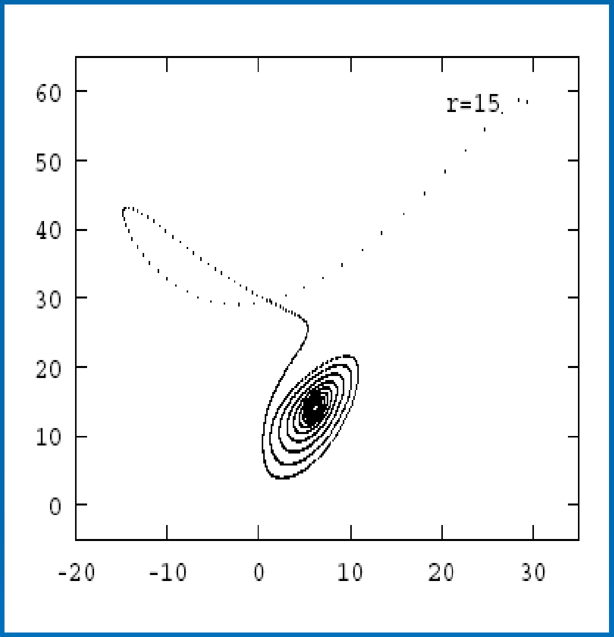


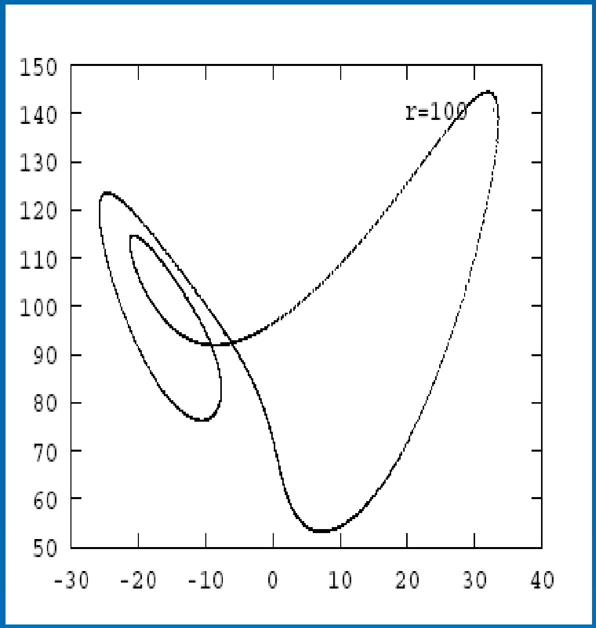
Phase space

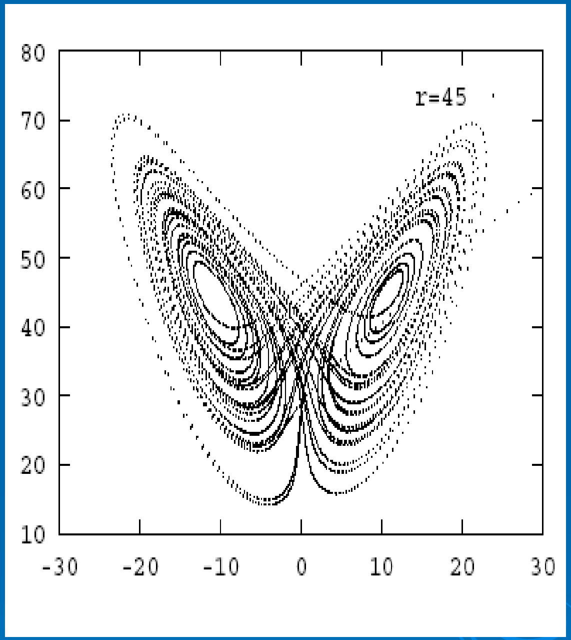


Example Lorentz (cont'd)

Phase space variation due to variation of the r parameter (2-D projections)







Point attractor

Periodic orbit

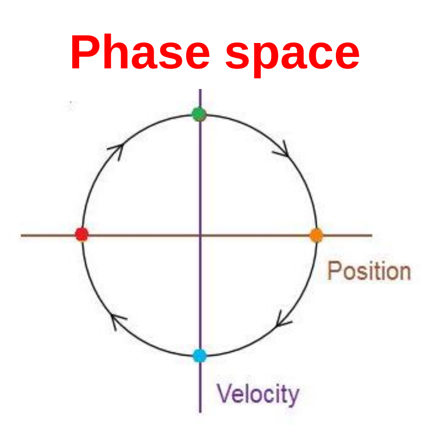
Chaotic (strange) attractor

Geometry can help detect transitions!

Utility of phase space

- It gives us an idea about all the allowed states of our system

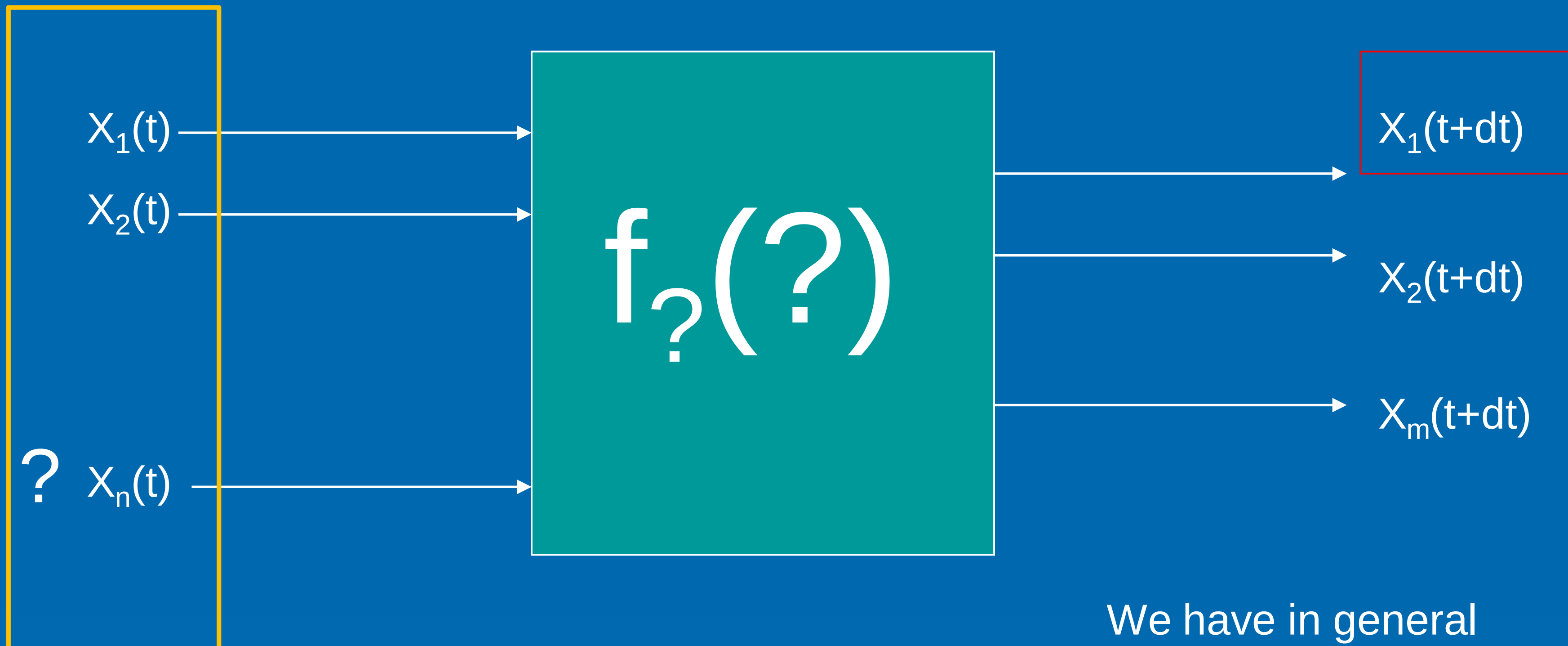
- Knowing one or more variables we can estimate the others



- It is a geometrical object → we can use geometrical tools to compare different systems or states

- if an attractor exist we do not need to explore the whole phase space

Dynamical System (Practically)



We do not know the number of variables that describe the system state

We do not know the laws that describe the system evolution

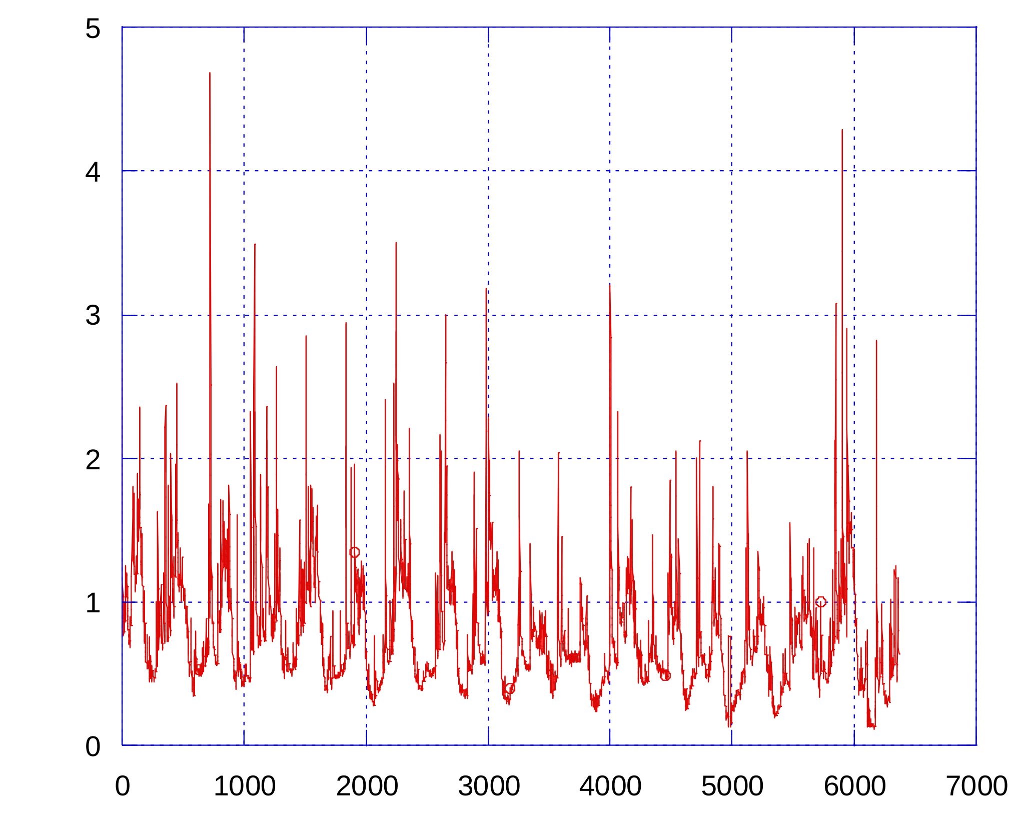
We have in general access only to several measurable quantities of the system observables

Several Practical Aspects

- The quantity we observe is a system variable or a function of several system's variables?
- In some cases yes, but not always.

- The various observed quantities are independent or interdependent?

Inverse problem
Known time series – unknown dynamical system
Example (1)



Water level of river Nestos (Northern Greece)
Temenos measurement Station : 01/01/1980 - 30/05/97

Questions

- Can we extract from one time series useful information about the simplest model system that can describe the real system ?
- The time series originates from a deterministic or stochastic system?
- The underlying system is linear or non-linear?

What can be the results of the analysis?

- Elements of system dynamics:

characteristic times/scales of the underlying system

- System Identification, Change of State
- Prediction of future values
- System Control

What can be the results of the analysis?

- Elements of system dynamics:

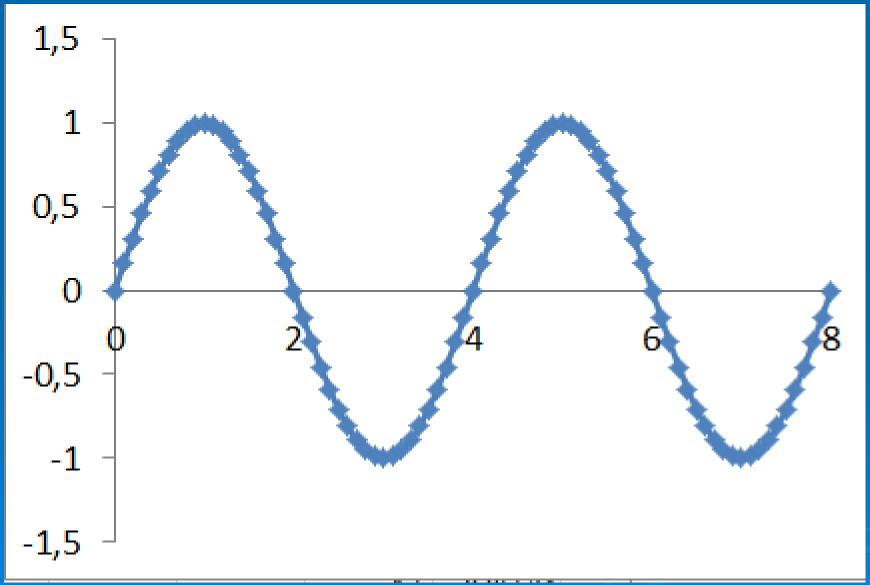
characteristic times/scales of the underlying system

- System Identification, Change of State
- Temporal correlations
- Prediction of future values
- System Control

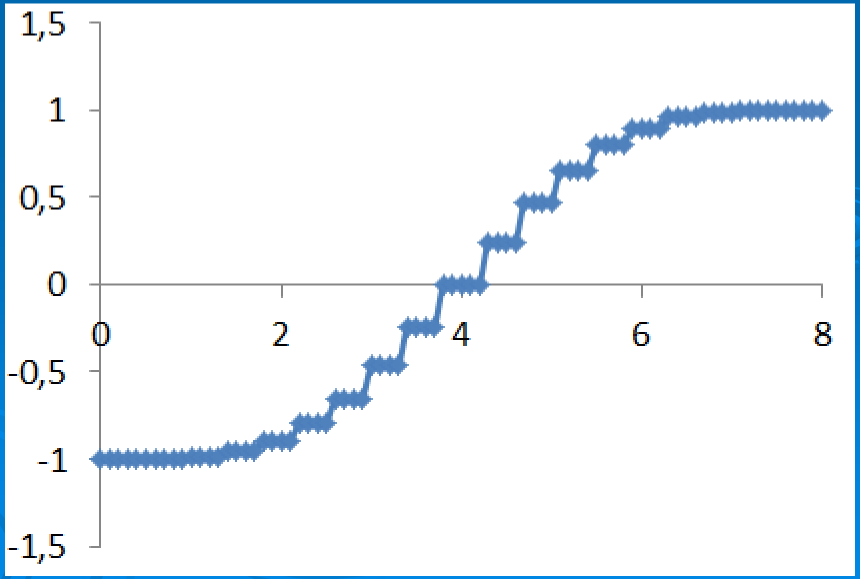
What you already know to do

Statistical analysis mean value, standard deviation, skewness, kyrtosis

The problem is that statistical analysis
gives the same results for different temporal variations



Just order the data →
same statistic but..



Temporal behavior - linear methods: regression analysis

Relation between successive measurements is linear

$$X(t+dt)=A*X(t)+B$$

Or dependence on several previous measurements

$$X(t+dt)=A_1*X(t)+A_2*X(t-dt)+\ldots+A_N*X(t-Ndt)+B$$

And for a system

$$X_1(t+dt)=A_{11}*X_1(t)+A_{12}*X_2(t)+\ldots+A_{N1}*X_N(t)+B_1$$

$$X_2(t+dt)=A_{21}*X_1(t)+A_{22}*X_2(t)+\ldots+A_{N2}*X_N(t)+B_2$$

……………………

$$X_N(t+dt)=A_{N1}*X_1(t)+A_{N2}*X_2(t)+\ldots+A_{NN}*X_N(t)+B_N$$

Unfortunately nature

is in general more complex

and in general non-linear

Time and Geometry
in asymptotic behavior of the system in time $\rightarrow \infty$

- **(a) convergence toward (or repellence from) given points of equilibrium** (points X where $f^{(1)}(X)=0$) depending on their state stable or unstable equilibrium.

- **(b) oscillations** that are **periodic**, if the trajectories are characterized by d frequencies $\omega_1, \omega_2, \ldots, \omega_d$ with rational ratio between them, or **nearly periodic** when frequencies present irrational ratios.

- **(c) chaotic behavior** кwhere the motion in the phase space is aperiodic , finite and is characterized by a continuous power spectrum and sensitive dependence on initial conditions.$P(f)=Af^b$

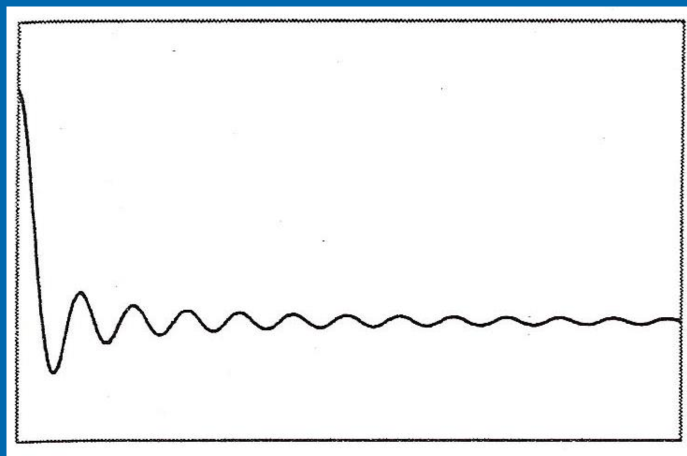
Attractors

- attractor : Set of points in the phase to which tend asymptotically all trajectories in the phase space for a range of initial conditions
- basin of attraction of an attractor: the set of initial conditions whose evolution leads to the attractor.
- The asymptotic form of the trajectory in phase space can be an attractor.

Point attractor

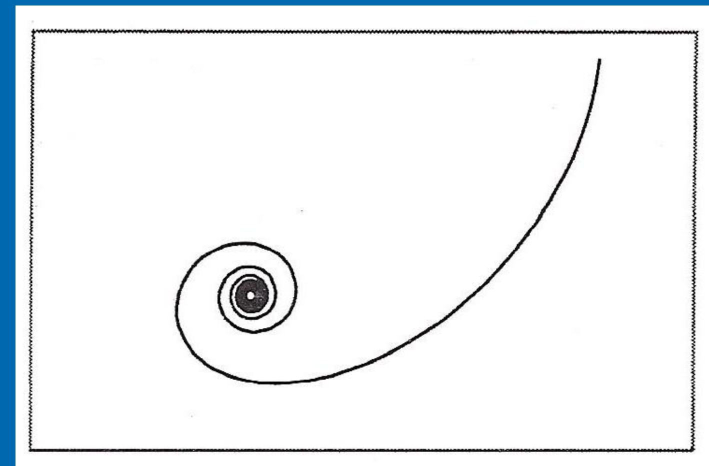
Time series

x



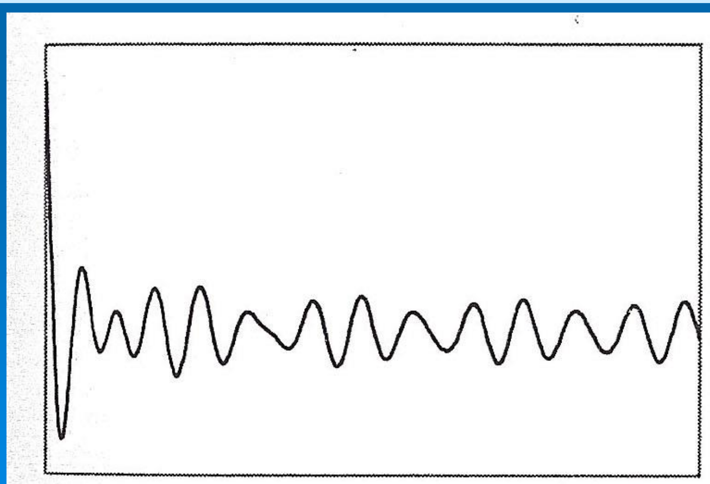
t

Phase space



Limit cycle attractor

x



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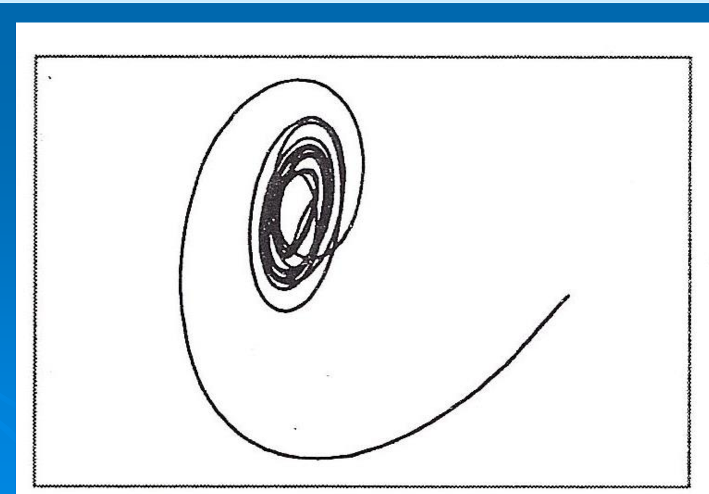
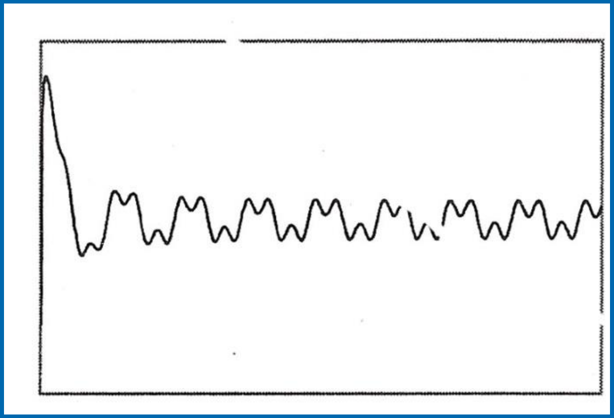


Toroidal attractor

Time series

x



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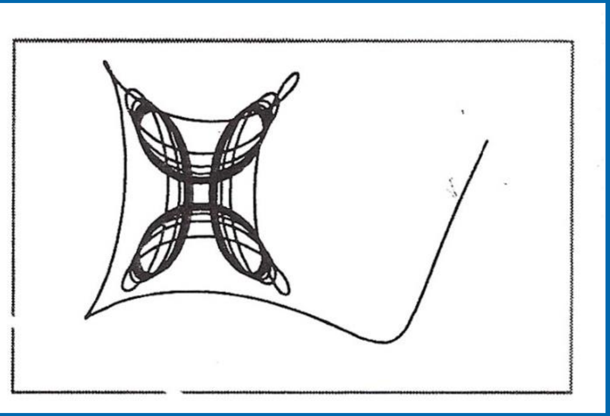
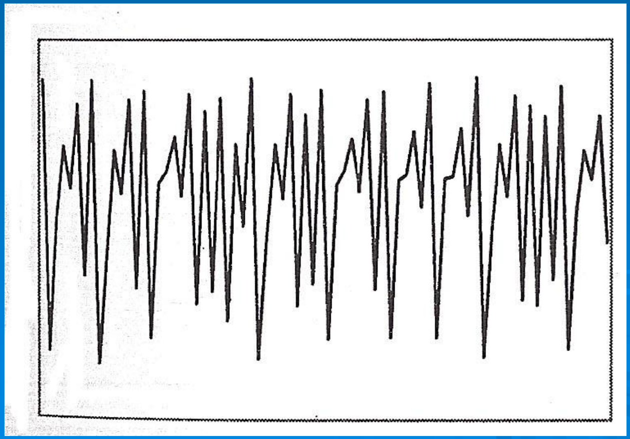


Atrractor

Chaotic or Strange attractor

x



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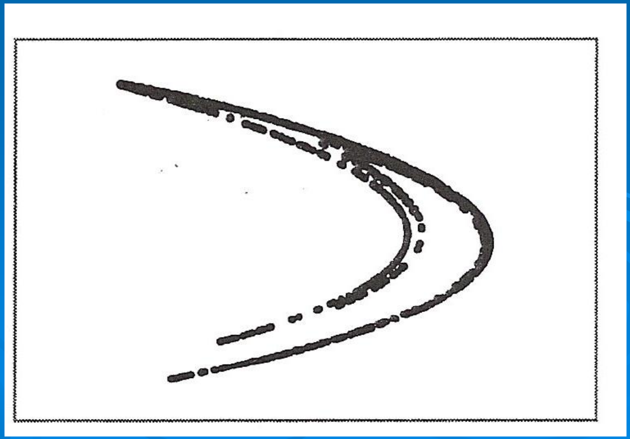

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LINEAR TOOLS

Autocorrelation function

$$R(\tau) = \frac{\frac{1}{N}\sum_{i=1}^{N}\left[x(i\Delta t+\tau)-\bar{x}\right]\left[x(i\Delta t)-\bar{x}\right]}{\frac{1}{N}\sum_{i=1}^{N}\left[x(i\Delta t)-\bar{x}\right]^2}$$

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x(i\Delta t)$$

Power Spectrum

$$P(f) = N\left\|A(f)\right\|^2$$

$$A(f) = \frac{1}{N}\sum_{j=1}^{N} x_j e^{\frac{i\,2\pi f\, j}{N}}$$

Transformation from time domain space to frequency domain

F(t) $\longrightarrow$ Φ(F)

Continuous Fourier Transformation

$$H(f) = \int_{-\infty}^{\infty} h(t) e^{2\pi i f t} dt$$

$$h(t) = \int_{-\infty}^{\infty} H(f) e^{-2\pi i f t} df$$

Discrete Fourier Transformation

$$H_n = \sum_{k=0}^{N-1} h_k e^{2\pi i k n / N}$$

$$h_k = \frac{1}{N} \sum_{n=0}^{N-1} H_n e^{-2\pi i k n / N}$$

Discrete Fourier Transformation (DFT)

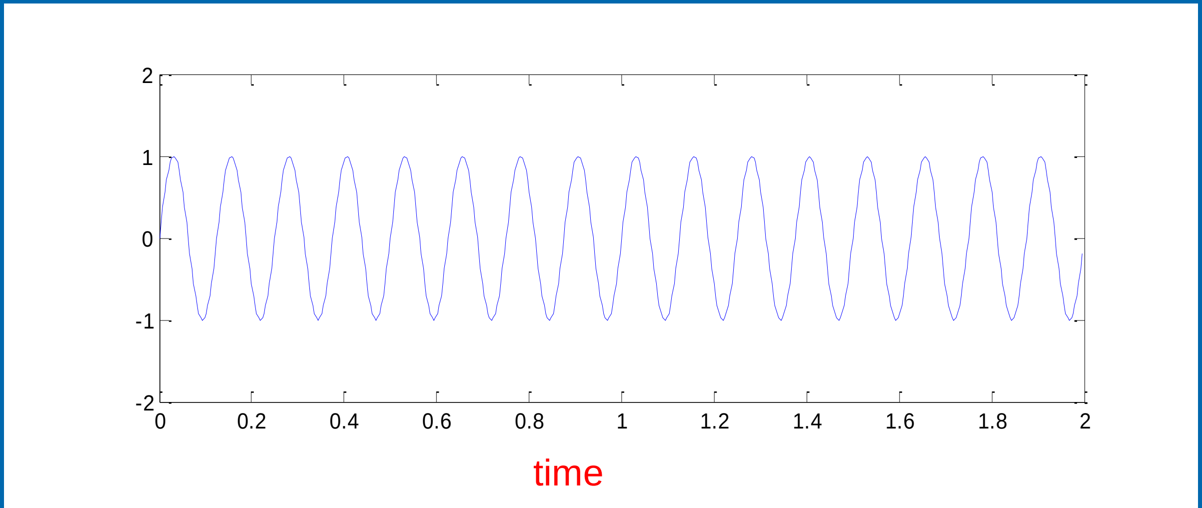
$$x_n \approx \frac{a_o}{2} + \sum_{m=1}^{N/2}\left[a_m \cos\left(\frac{2\pi mn}{N}\right) + b_m \sin\left(\frac{2\pi mn}{N}\right)\right]$$

$$a_m = \frac{2}{N}\sum_{n=1}^{N} x_n \cos\left(\frac{2\pi mn}{N}\right)$$

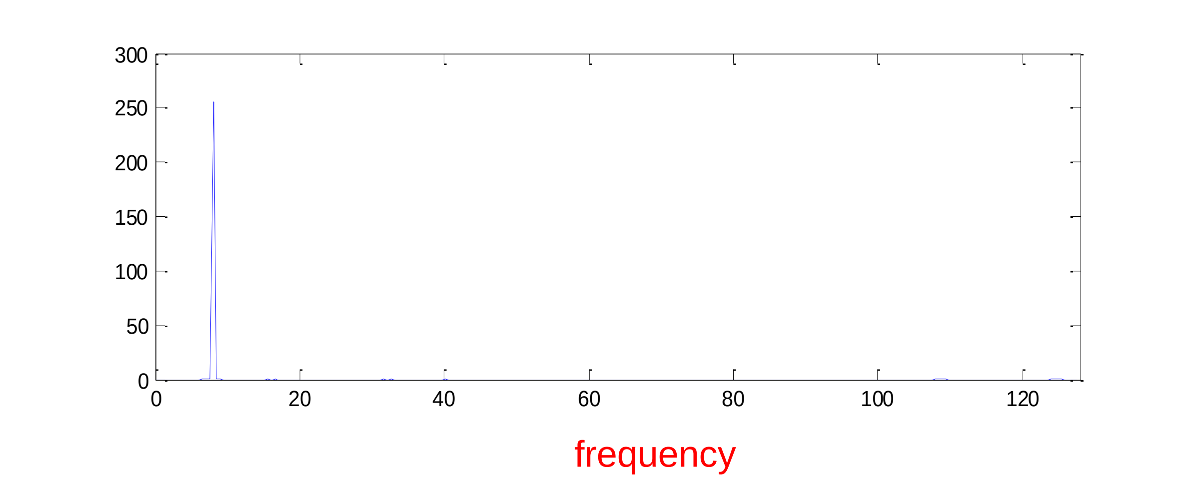
$$b_m = \frac{2}{N}\sum_{n=1}^{N} x_n \sin\left(\frac{2\pi mn}{N}\right)$$

$$S_m = a_m^2 + b_m^2$$ Power Spectrum

Example 1

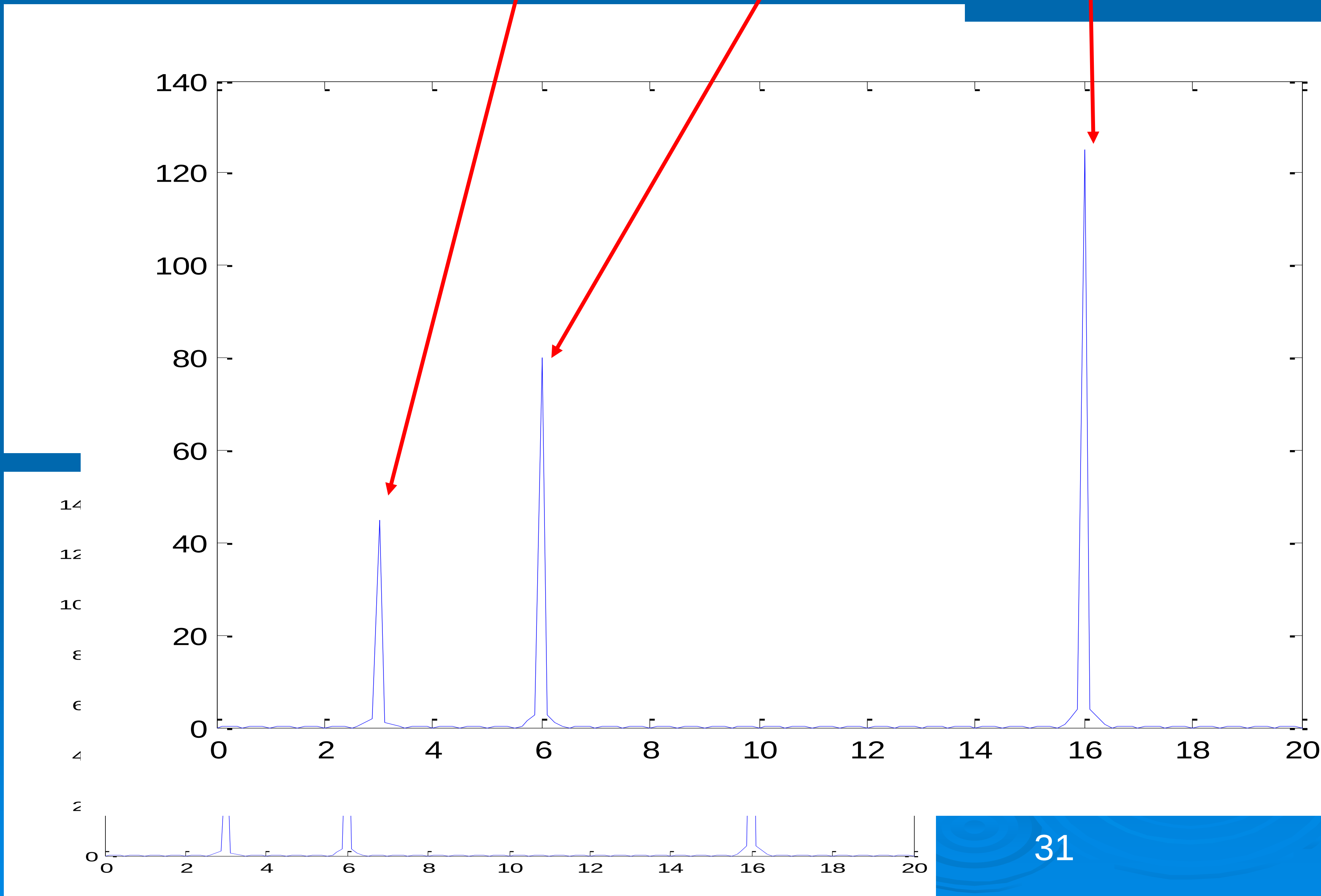


Initial function:
Sine wave



Fourier Transform
Delta function

$$y=3*\sin(2\pi 3t)+4*\sin(2\pi 6t)+5*\sin(2\pi 16t)$$



Utility of Fourier Transform

- Find Characteristic Frequencies i.e. times
- Extract periodic or seasonal components
- In many cases we can distinguish a near periodic time series from a chaotic one.

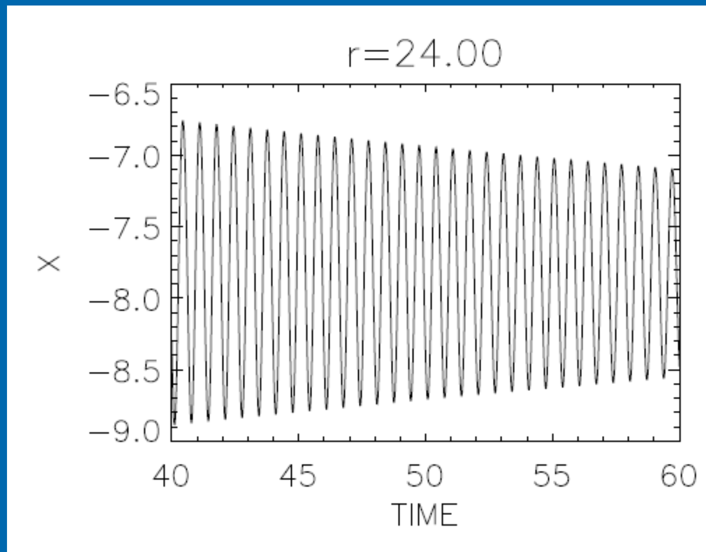
Power spectrum and Power law behavior

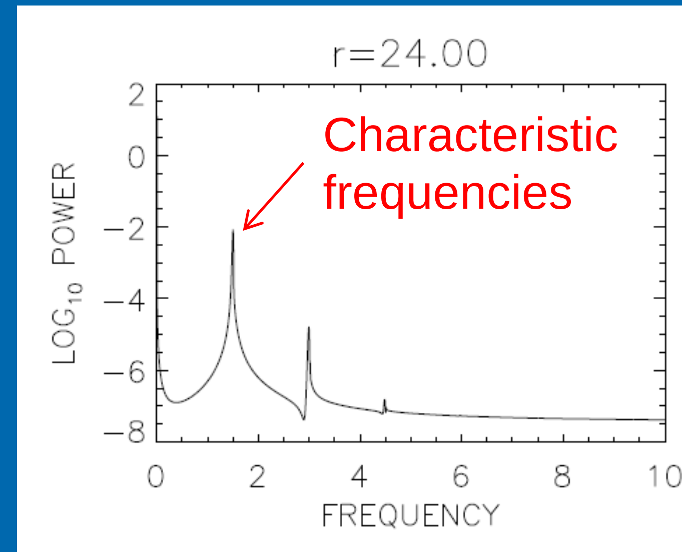
For several phenomena

- $P(f)=A*f^{\beta}$
- $\beta=1$ → Self Organized Criticality (Bak 1992)
- Variation of the value of the exponent β is an indicator of the change of the system state
- The importance of this behavior :
  there is not one ore more specific frequencies but a continuous range with variable contribution

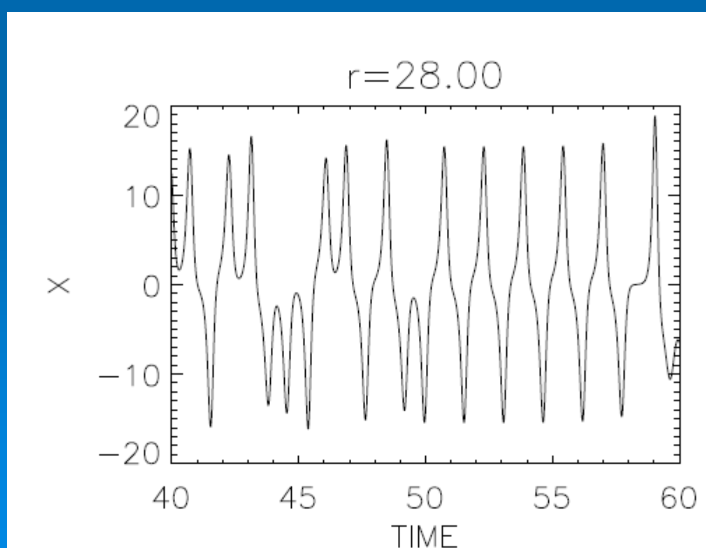
Power spectrum example: Lorenz system

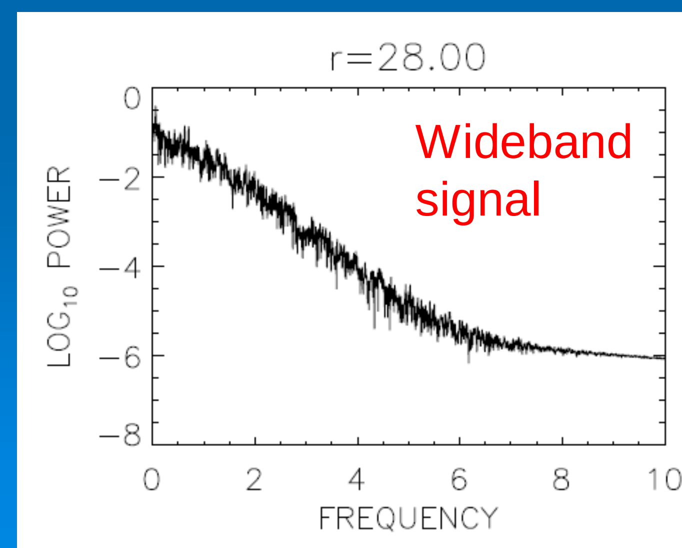
Non chaos





Chaos





AND NOW

SOME APPLICATIONS

Application : rainfall

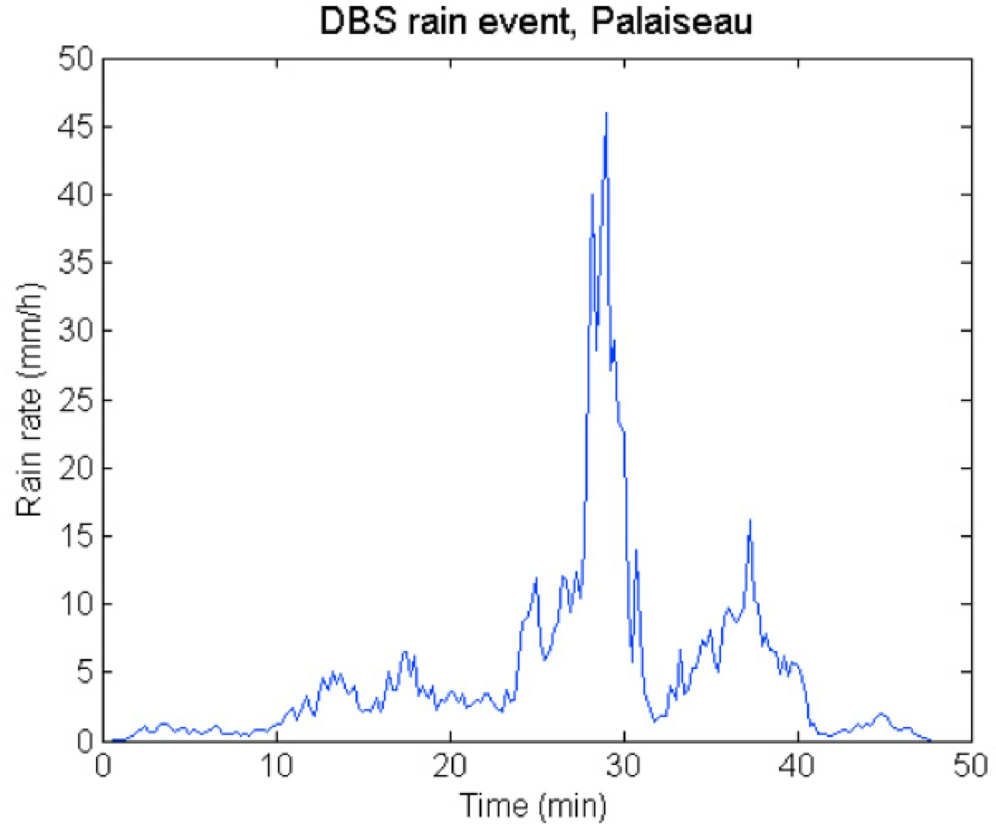
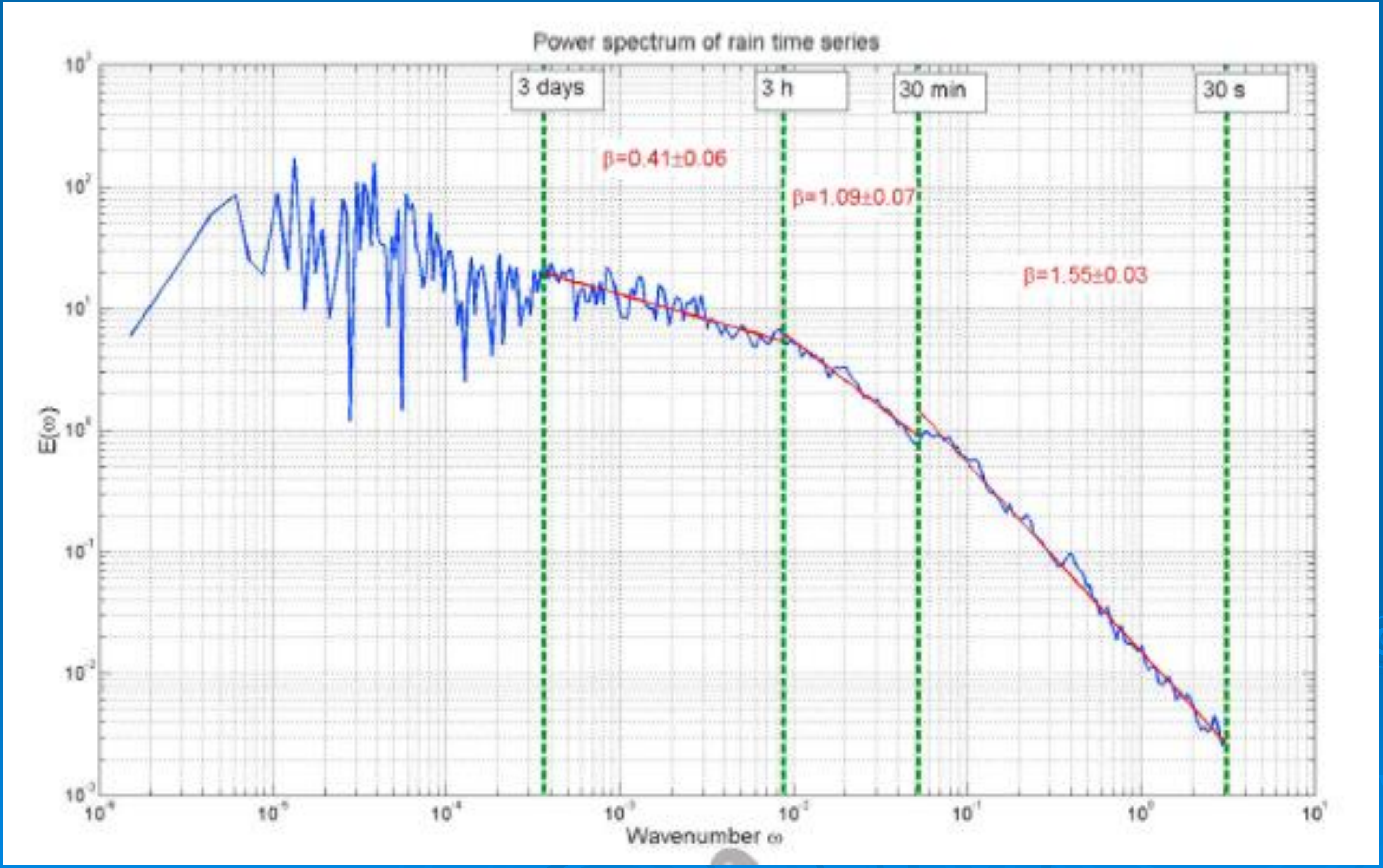


Figure 1. Rain event observed by the DBS on 12 May 2009, beginning at 04 h 05, in Palaiseau.

(Verrier et al 2011)

Rainfall power spectrum $P(f)=A/f^{\beta}$



Temporal analysis: Hurst Exponent

If *T(i), i=1,...,N*, is a time series T.
For every n, 2≤n≤N, we denote by $<T>_n$ the mean value of the first n elements of the time series T, i.e.,

$$<T>_n = \frac{1}{n}\sum_{j=1}^{n} T(j)$$

Then compute Xi(n), the accumulated deviation from the mean <T>n

$$X_i(n) = \sum_{k=1}^{i}\left[T(k) - <T>_n\right]$$

The range of the accumulated deviation from the average level is the difference between the maximum and the minimum cumulative deviations over n periods and is denoted by *Rn*, i.e.,

$$R_n = \max_{1\le j\le n}\left(X_j(n)\right) - \min_{1\le j\le n}\left(X_j(n)\right)$$

Hurst Exponent (cont'd)

- The function *Rn* generally increases as a function of n. Let us also denote by *Sn* the standard deviation of the first n elements of the time series *{T(j)}, i.e.,*

$$S_n = \sqrt{\frac{1}{n}\sum_{k=1}^{n}\left[T(k) - \langle T \rangle_n\right]^2}$$

In the case of a fractional Brownian motion in the limit of large n,

$$(R_n / S_n) \propto (n)^H$$

with 0≤H≤1, the Hurst exponent.

We can plot log(*Rn/Sn*) *vs*. log(*n*) and, in the case of scaling, estimate the value of the Hurst exponent.

Significance of Hurst exponent

- For time series with consecutive values generated by statistically independent processes with finite variances $H=0.5$ (uncorrelated or white noise).

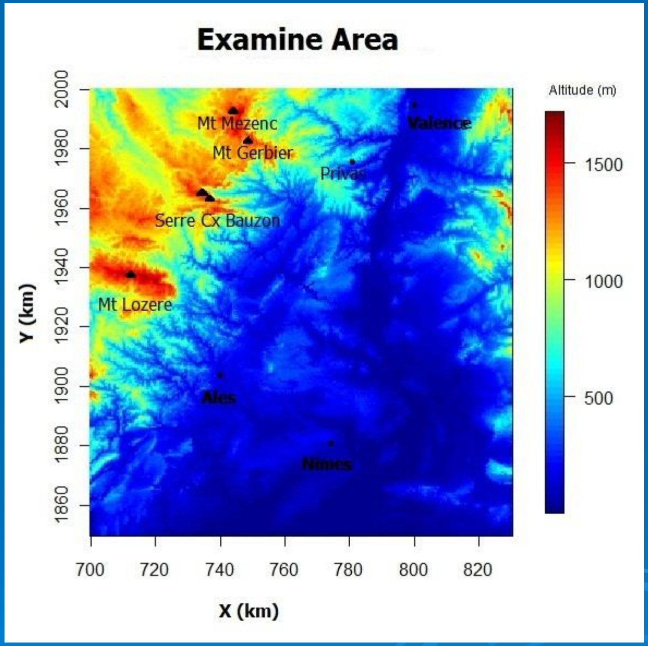
- $0.5<H<1$ processes where fluctuations in subsequent values are positively correlated (persistence), e.g. large positive values are followed by large positive values (or large values) and vice versa.

- $0<H<0.5$ corresponds to processes where fluctuations in subsequent values are negatively correlated (anti-persistence) e.g. large positive values are followed by large negative values (or very small values) and vice versa.

Hurst exponent application

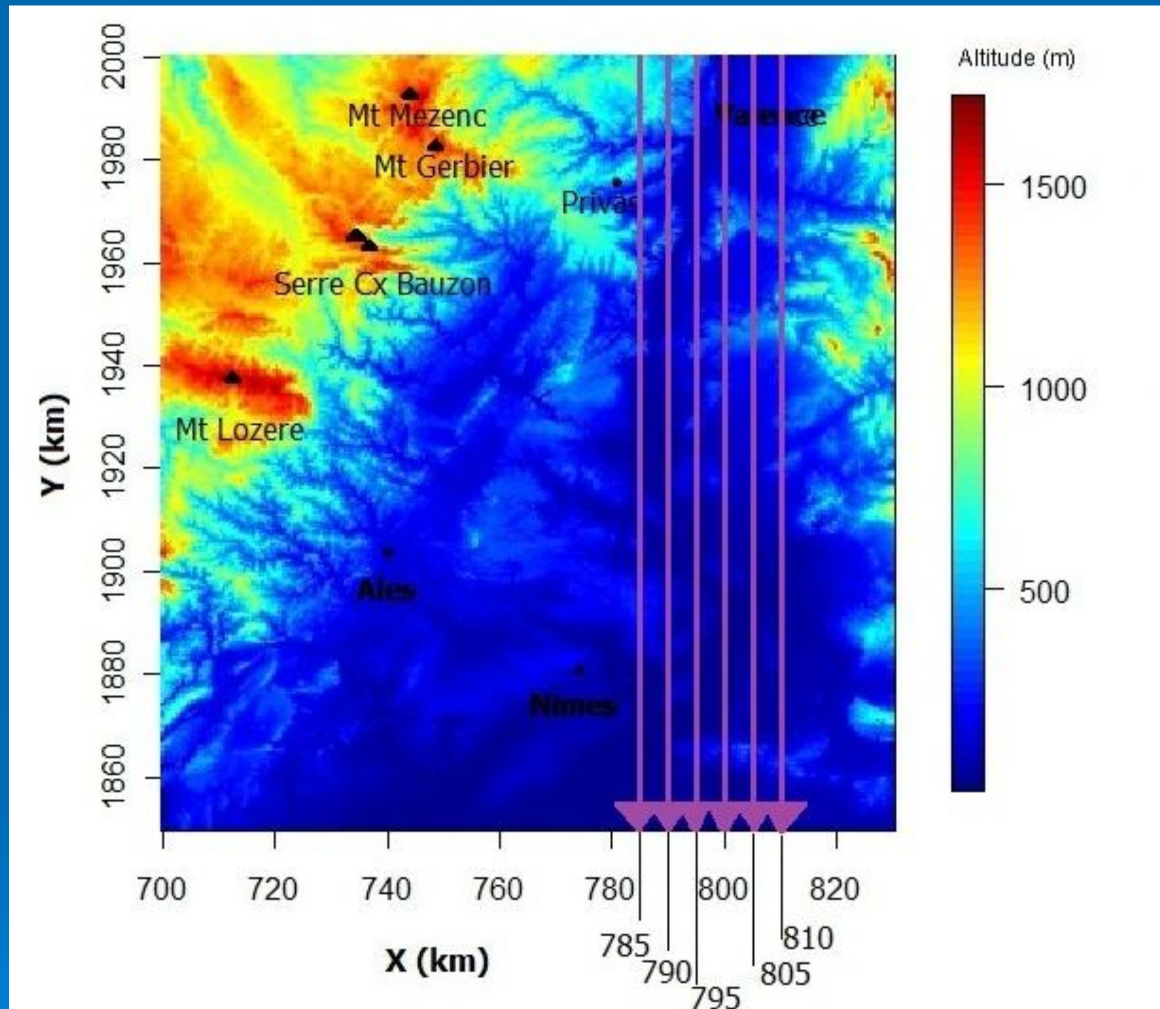
- Analysis of rainfall radar data

- Analyse the persistent and chaotic character of rainfall over Rhône river valley.



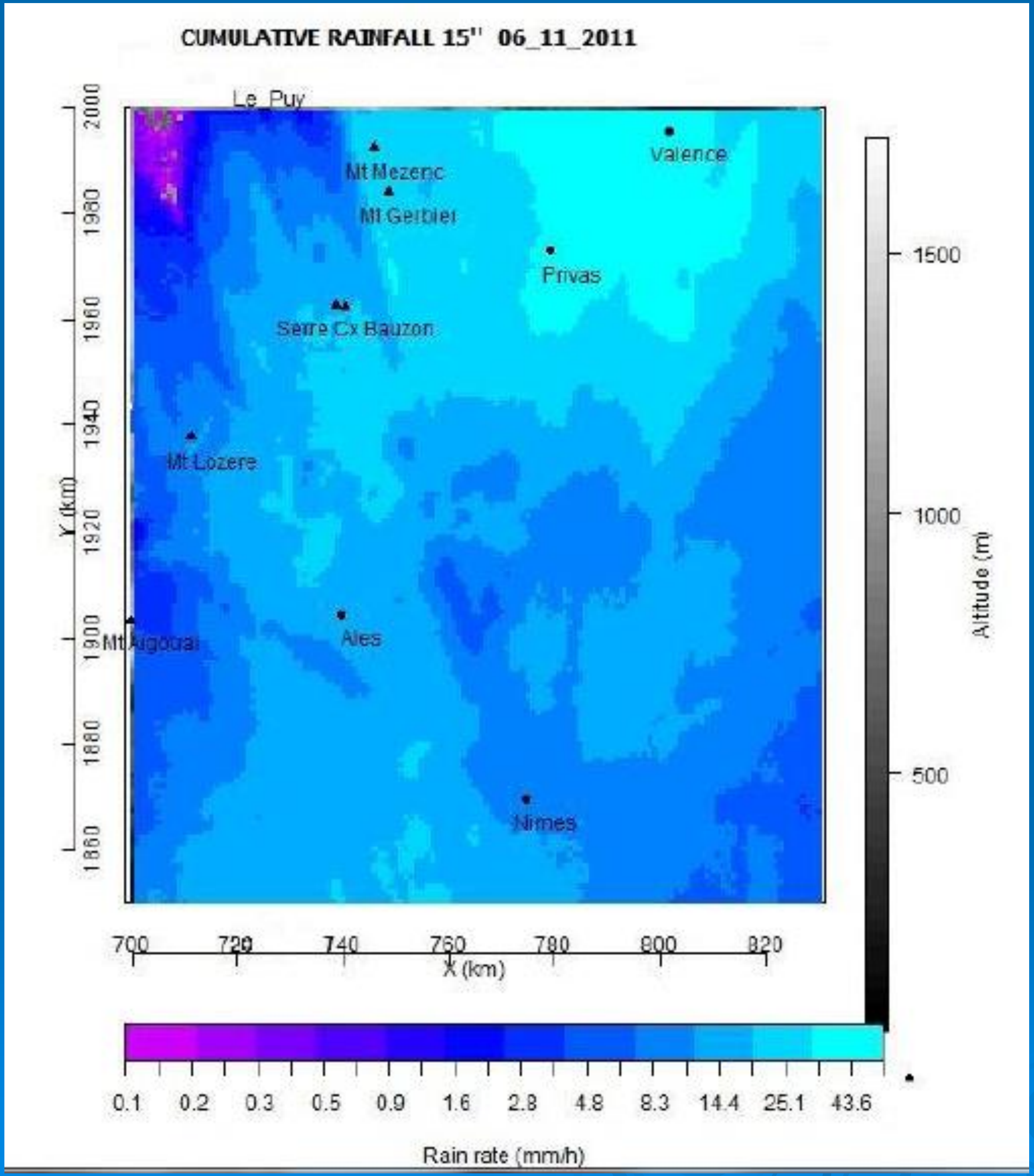
The role of the non-linear relief-rain interaction in the rainfall intensity structure. Molinie, G. Karakasidis, T., Triantafyllou, A. Creutin, J-D., Anquetin, S., EGU General Assembly 7-12 April, 2013 Vienna,

Examine area cross sections



Vertical cross sections above Rhone river valley
Resolution of 1km x 1km

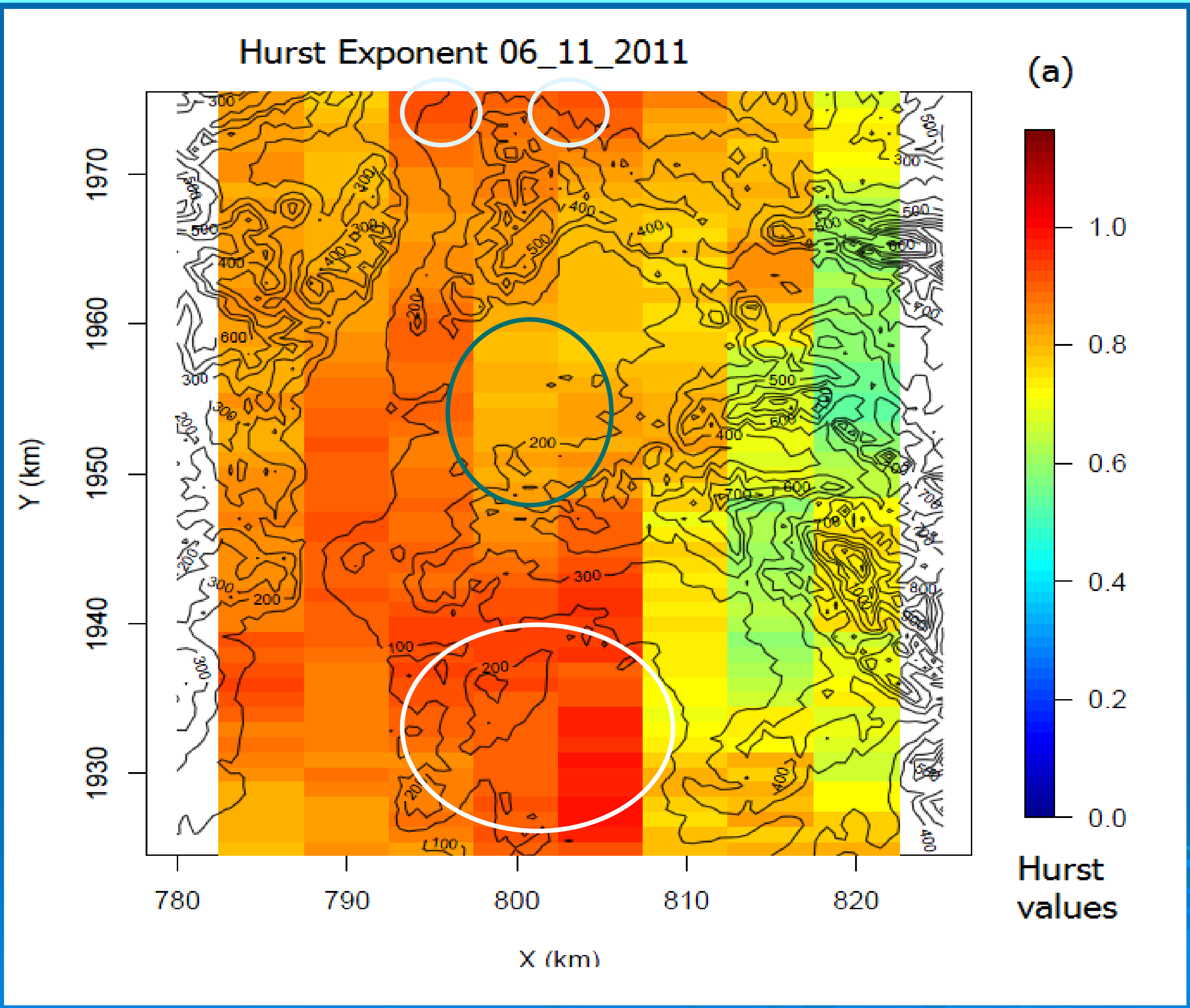
Radar Image



Resolution 1km x 1km

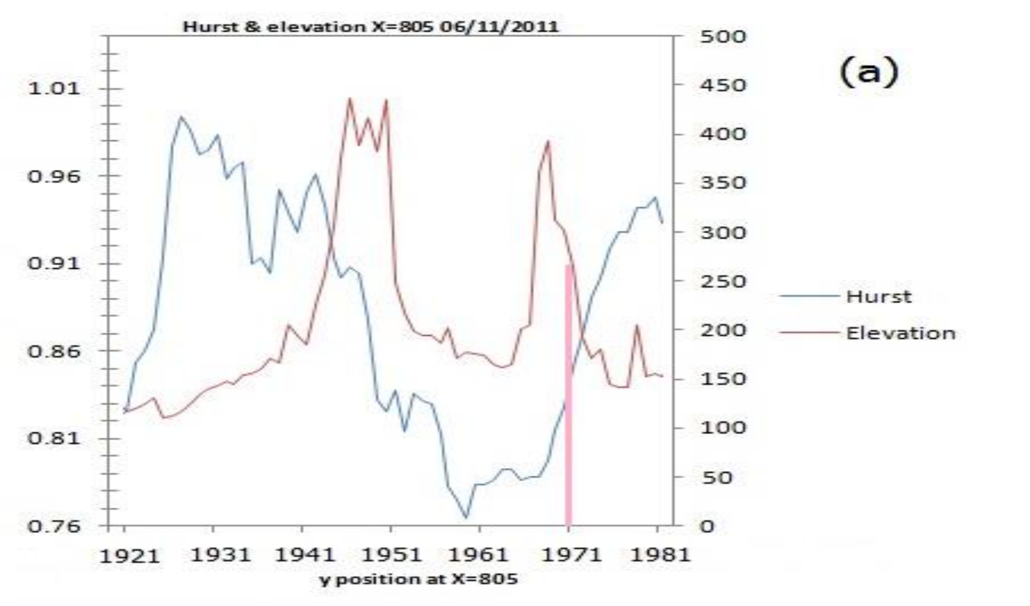
150 rain time series
At each vertical cut

Hurst map



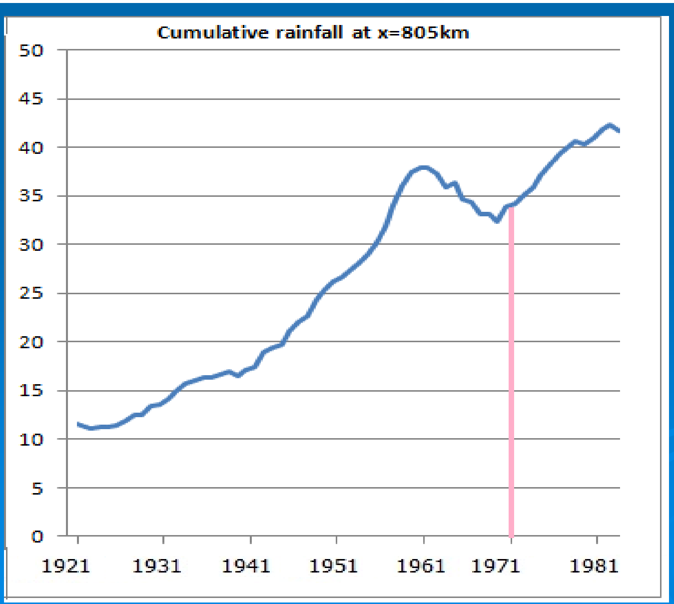
Mountains seems to block persistence and complexity

Cross section at X=805km



Between the mountain crests Hurst Exponent shows the lowest values in this event.

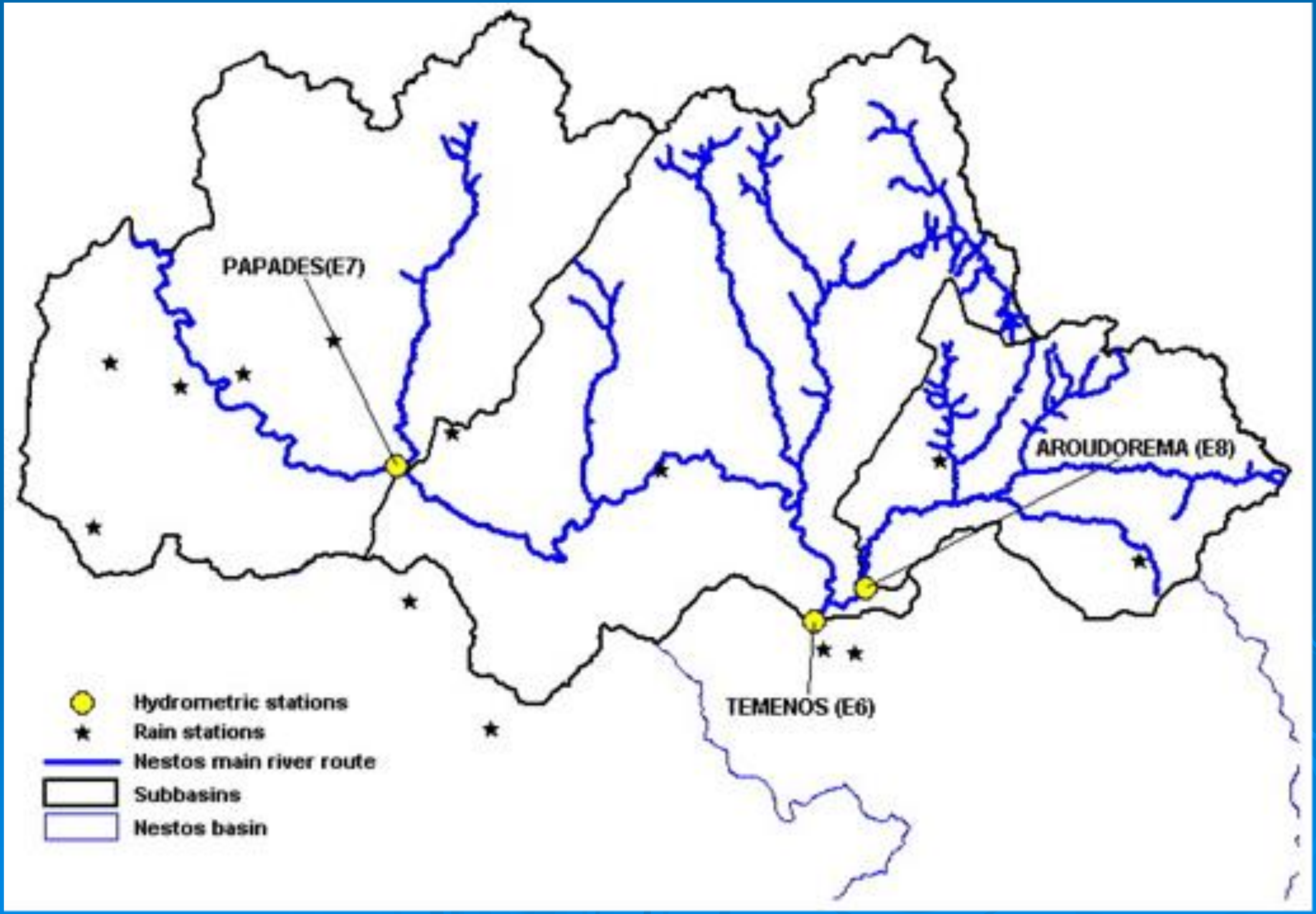
Cumulative rainfall increases almost above the first big mountain range (its natural due to turbulence presence)



The relief change has an influence on Hurst parameters

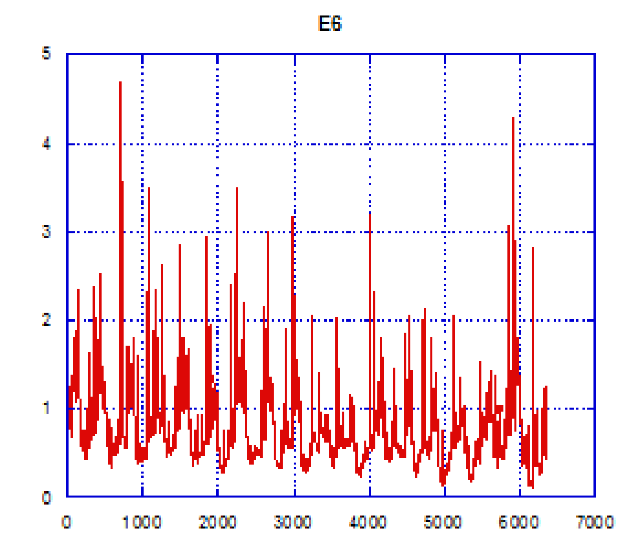
Hurst parameter behaviour reveal that the foothill presence creates a perturbation in rainfall structure; because of the variation of height rather than the height itself

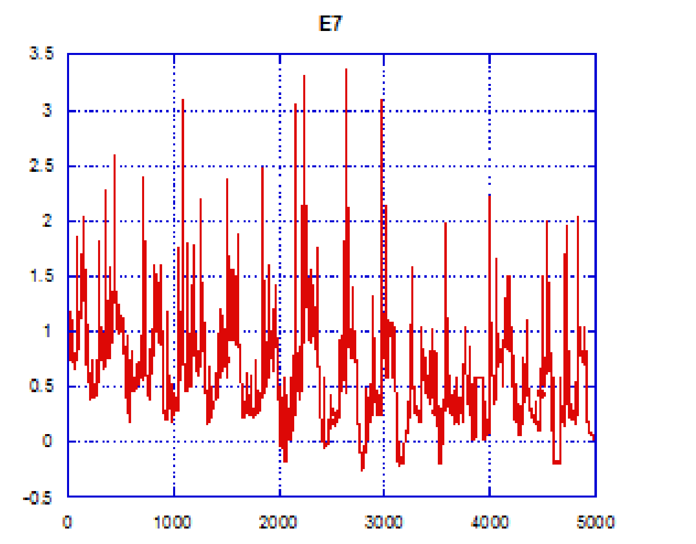
Application: identify different spatial behavior:
water level of river Nestos



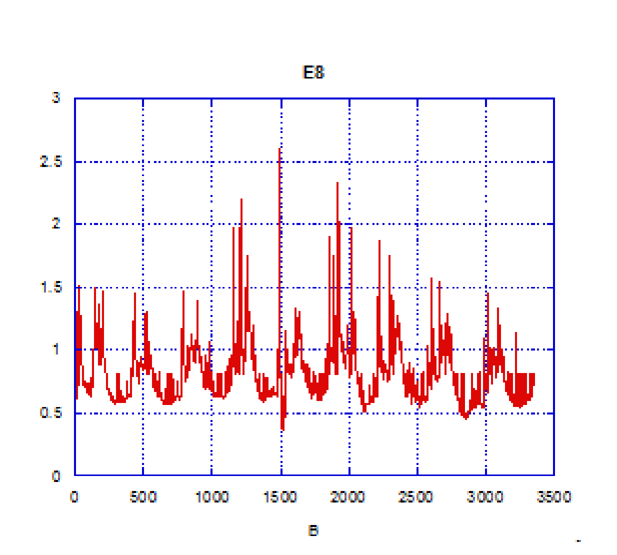
River water level time series at three positions

H=0.77





H=0.83

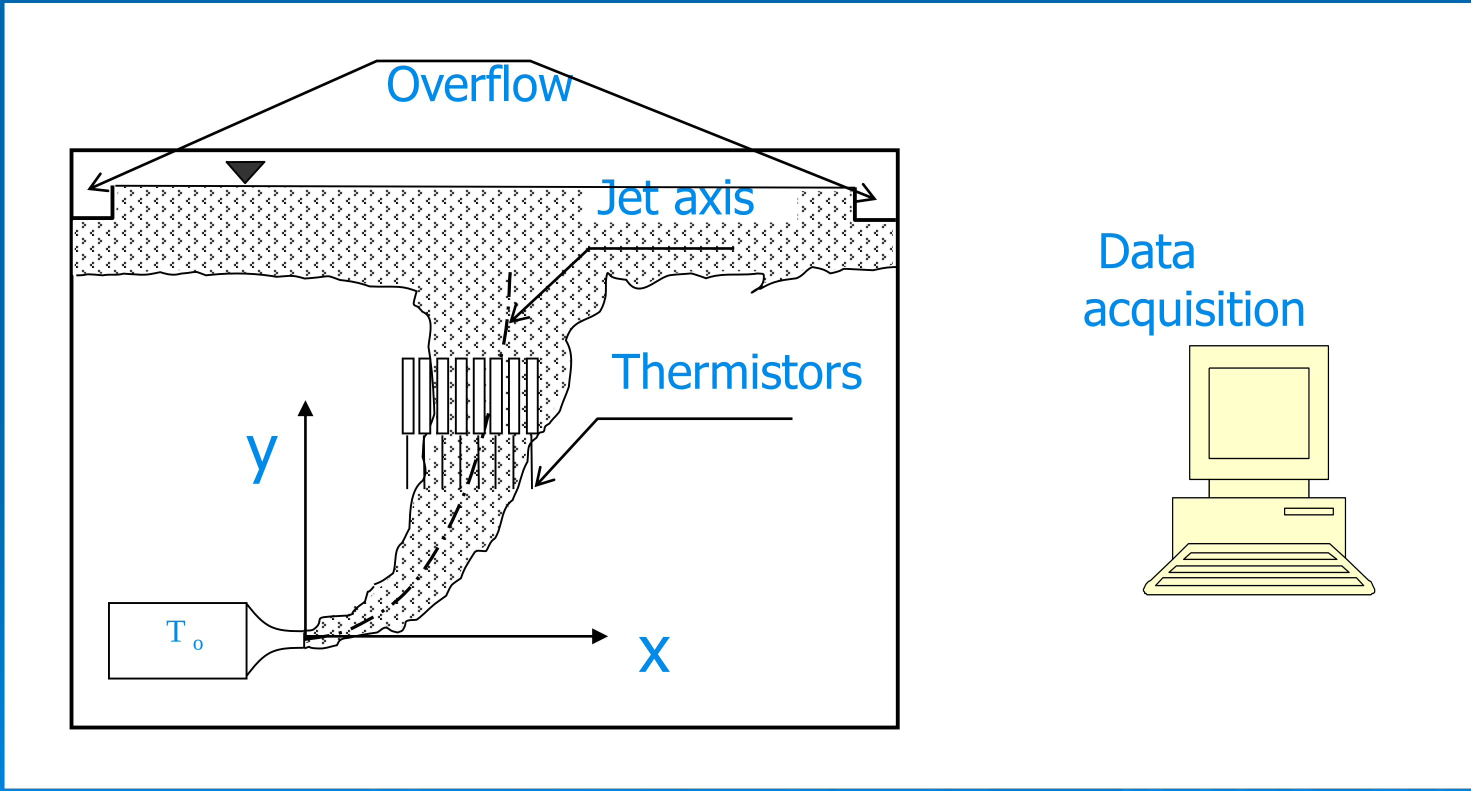


H=0.465

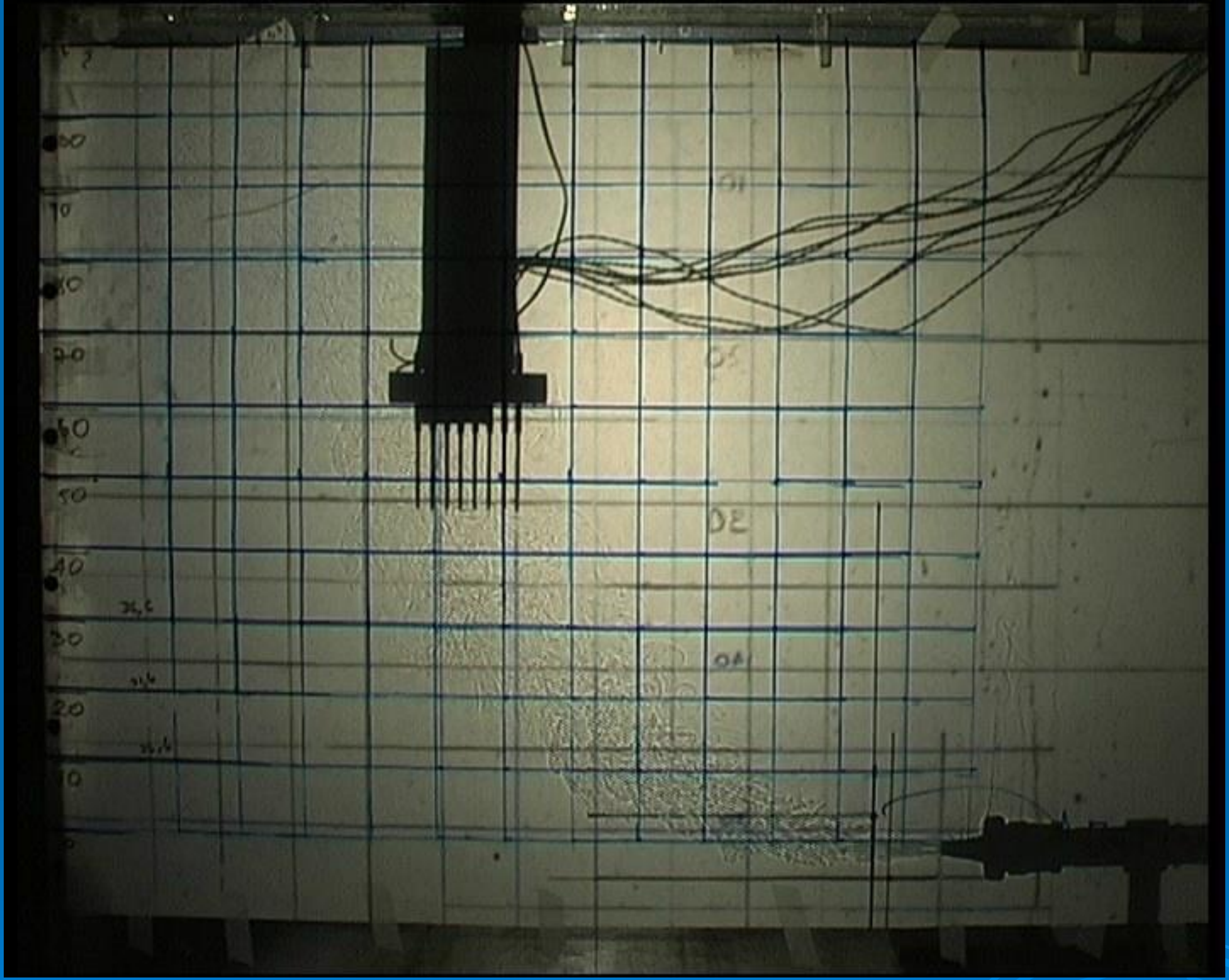
The similarities were found to be linked to similarities in the ground characteristics (slopes, materials)

Application in spatiotemporal variation Macroscopic Fluid Flow

Time-series analysis of temperature fluctuations in a horizontal round heated jet.



The ambient water temperature varied between 18-20°C, while the jet water temperature was around 60°C. A jet nozzle of 0.65cm diameter was used.



Question :

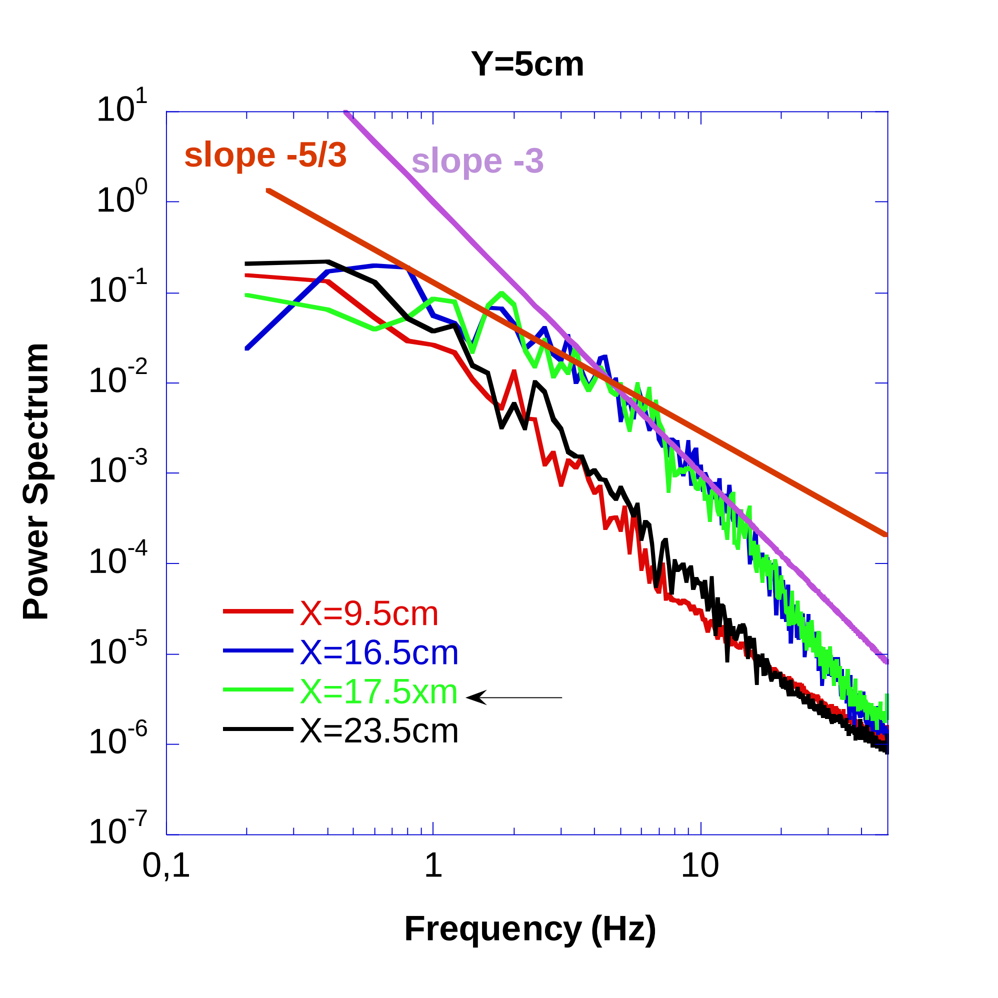
where the jet axis is located since the boundaries are not well defined ?

Horizontal round heated jet: observables

Instantaneous temperature time series were recorded at several points across a horizontal line of the jet as a function of their distance from the center line.

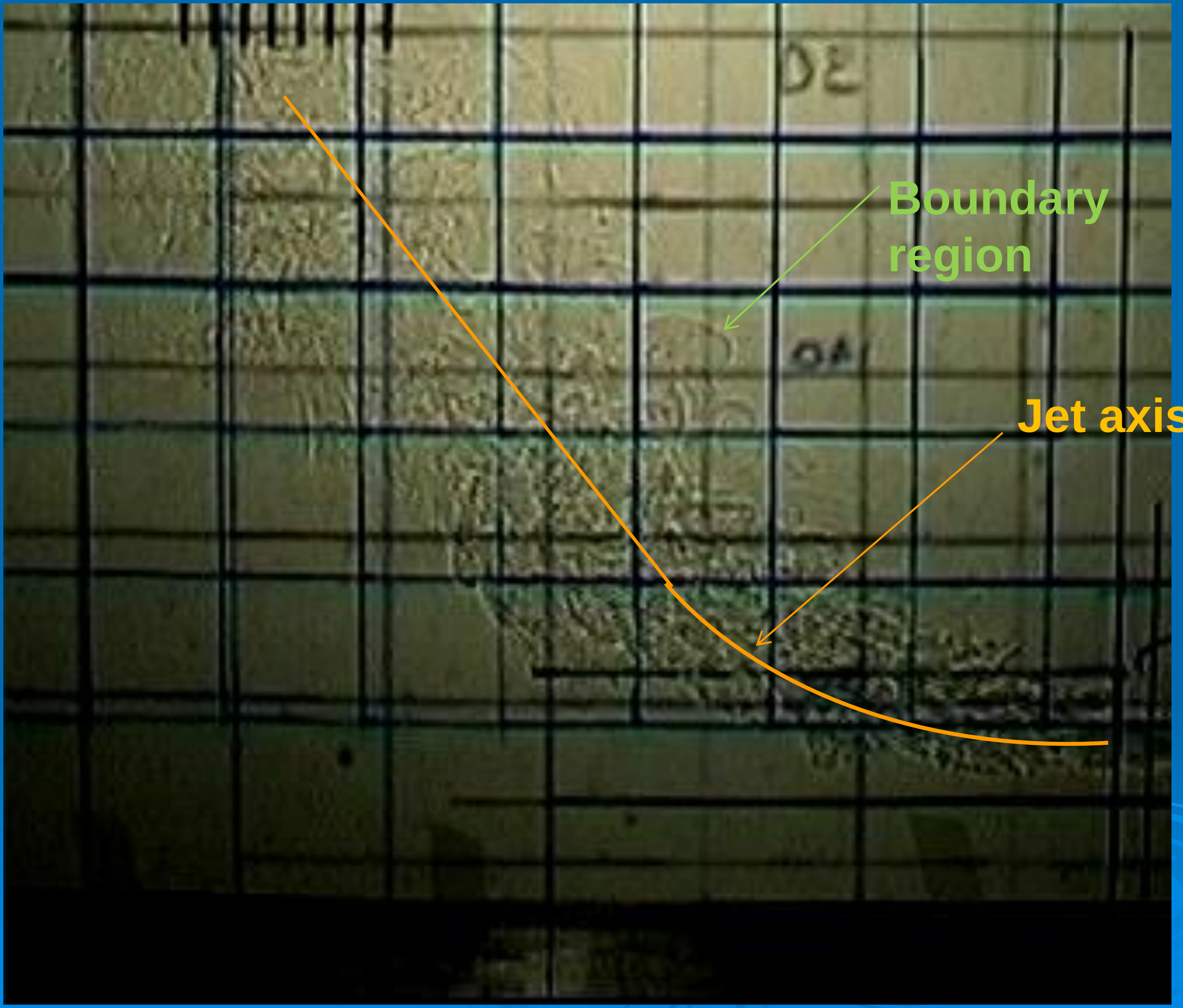
The variation of the characteristic time scales obtained from the above analysis is associated with and interpreted via the transitions of the physical state of the flow from the center line to the boundary of the jet.

Turbulent fluctuations: Power Spectrum



a narrow but distinct region where we have approximately conditions of fully developed homogeneous turbulence.

another region at the higher frequencies of the spectrum with higher slope (close to 3). Analogous behavior have been observed in vertical jet flow [Papanicolaou et al. 1987]


Boundary
region
Jet axis

Non linear Temporal Analysis: Average Mutual information (AMI)

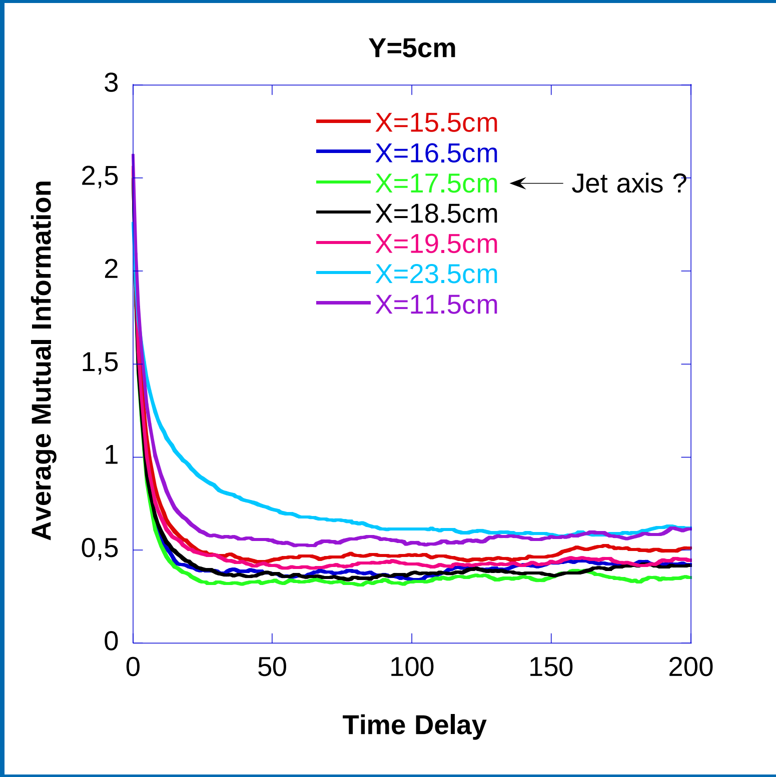
- Definition

$$I(X,Y)=\sum_{x,y} p_{XY}(x,y)\log\frac{p_{XY}(x,y)}{p_X(x)\,p_Y(y)}$$

- $p_X(x)$ the probability that X=x, $p_Y(y)$ the probability that Y=y
- $p_{XY}(x,y)$ the joint probability X=x and Y=y

- The AMI takes only positive values.
- We choose as time delay the time corresponding to the first local minimum of AMI (there correlations become small for the first time)
- AMI takes into account not only linear correlations between successive measurements

Application in turbulent jet



Close to the jet centerline memory is lost fast. At locations far from the centerline memory is large.

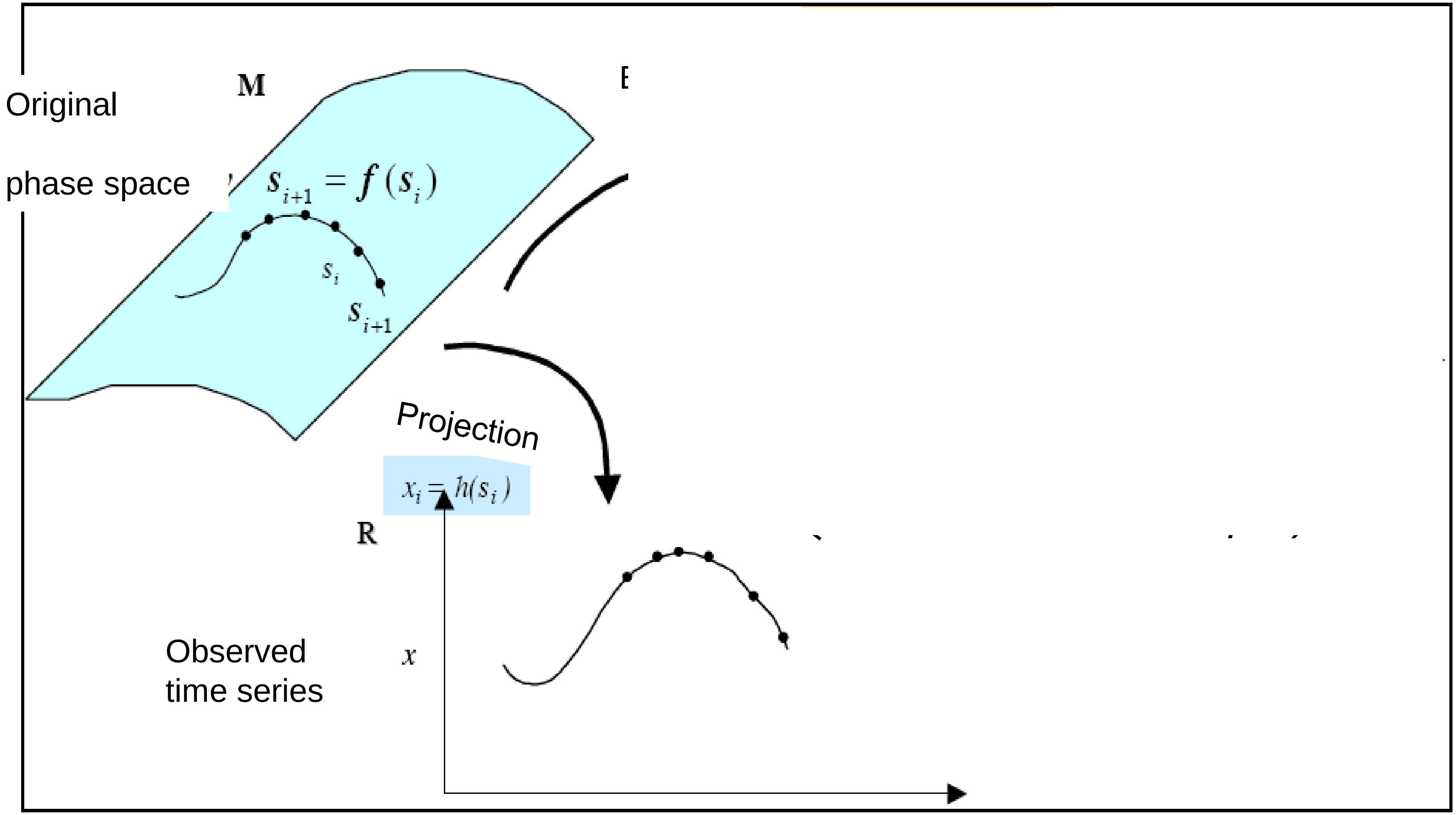
Short memory corresponds to short living flow structures - in the axis region we expect to have fully developed turbulence.

As we go towards the ambient fluid, larger scale flow structures dominate the flow. These flow structures live longer and give rise to longer memory effects.

Plan of Presentation

- Introduction to time series
  - Linear Methods
  - Non Linear Methods
- Temporal behavior
  - Power spectrum
  - Hurst exponent
  - Average mutual information
- Phase space reconstruction methods
  - Recurrence plots
- Applications in field measurements
  - turbulent heated jet
  - river water level
  - Rainfall
- Conclusions

Phase space reconstruction



Kugiumtzis 1999

The invariant quantities of the two spaces must remain the same

Example of Phase space: The Lorenz System

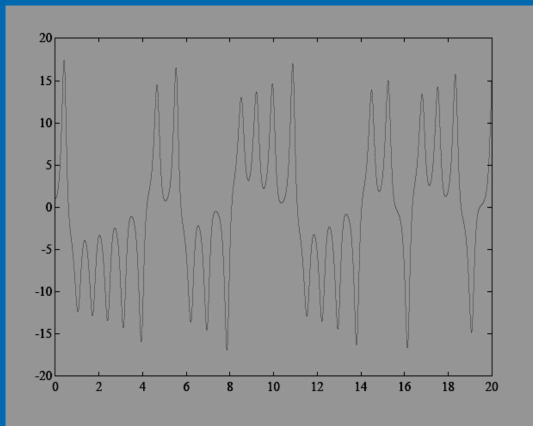
$$\frac{dx_1}{dt} = \sigma(x_2 - x_1) \qquad \frac{dx_2}{dt} = rx_1 - x_2 - x_1x_3 \qquad \frac{dx_3}{dt} = x_1x_2 - bx_3$$

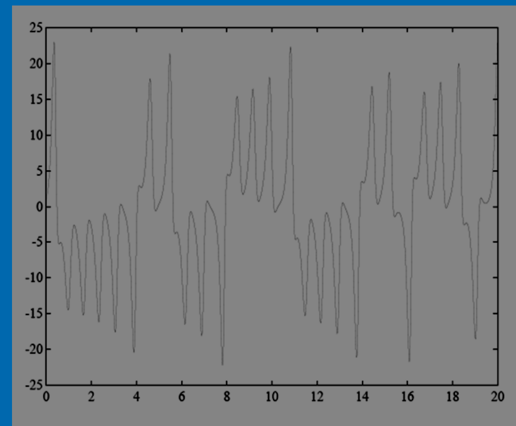
x_1 proportional to the intensity of the convective motion,
x_2 proportional to the temperature difference between ascending and descending currents,
x_3 proportional to distortion of vertical temperature profile from linearity.

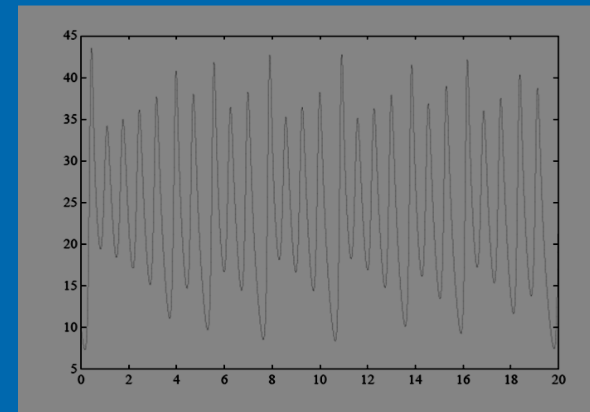
- σ Prandtl number
- r normalized Rayleigh number
- b gives the size of the region approximated by the system

For values σ=10 r=28 και b=8/3 the system becomes chaotic, i.e. a small change in initial conditions can result in a large change in the output.

$x_1(t)$ $x_2(t)$ $x_3(t)$

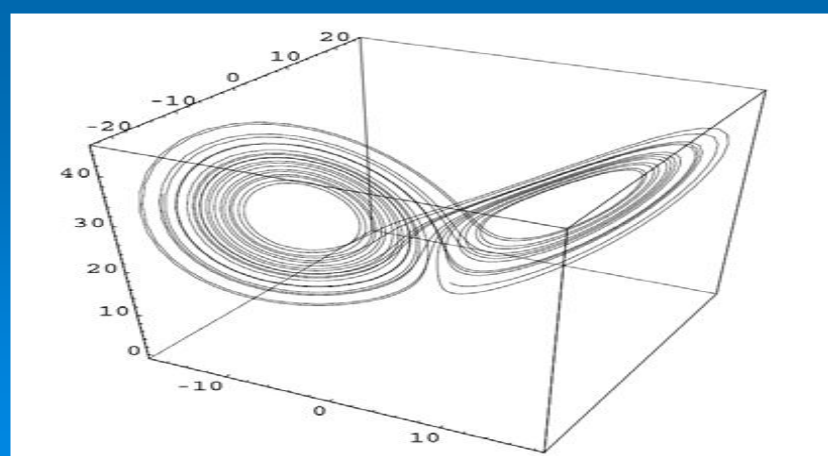






For every time t plot all points (x_1, x_2, x_3)

Phase space of the Lorenz system



Phase Space Reconstruction (2)

- We are interested in
  metric (topological) properties such as the dimension of the attractor

- It is not necessary to reconstruct the full phase space given that in the majority of cases the system presents an attractor with smaller dimension

- The reconstructed phase space has the following properties
  - Each point is mapped through the dynamics to a unique successive point
  - There is a smooth and nonsingular transformation between the reconstructed space and the original space.

  - This methodology was introduced by Packard et al (1980) and Takens (1981)

Phase space reconstruction (1)

- If the system is deterministic it is reasonable to expect that each measurement depends on a given number of previous measurements, i.e. $X_{n+1}=f(X_n, X_{n-1}, X_{n-2}, \ldots, X_{n-m})$
- and that for small Δt these values include information equivalent to that of several derivatives that could be described by the system evolution

$$\frac{df}{dt} \approx \frac{X_n - X_{n-1}}{\Delta t}$$

$$\frac{d^2 f}{dt^2} \approx \frac{X_n - 2X_{n-1} + X_{n-2}}{(\Delta t)^2}$$

$$\frac{d^3 f}{dt^3} \approx \frac{X_n - 3X_{n-1} + 3X_{n-2} - X_{n-3}}{(\Delta t)^3}$$

$$\frac{d^4 f}{dt^4} \approx \frac{X_n - 4X_{n-1} + 6X_{n-2} - 4X_{n-3} + X_{n-4}}{(\Delta t)^4}$$

Method of Delays

- The reconstruction of the phase space is done via the construction of a m-dimensional vector states S_i from the time series in the following way:

$$S_i = \left[x_i, x_{i+\tau}, x_{i+2\tau}, \ldots, x_{i+(m-1)\tau} \right]$$

- Parameters necessary for the reconstruction
  - A) embedding dimension m (in what space I plot my points)
  - B) time delay τ (how close the timeseries points are)

Choice of time delay τ

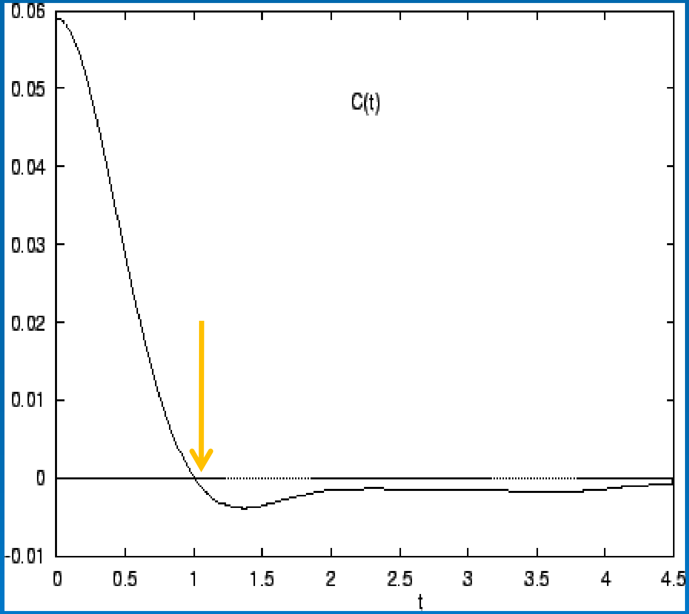
- If τ is too small then successive time series values are unnecessarily strongly correlated and the chosen components do not contain additional information
- If τ is too large the values of the time series may become unnecessarily extremely

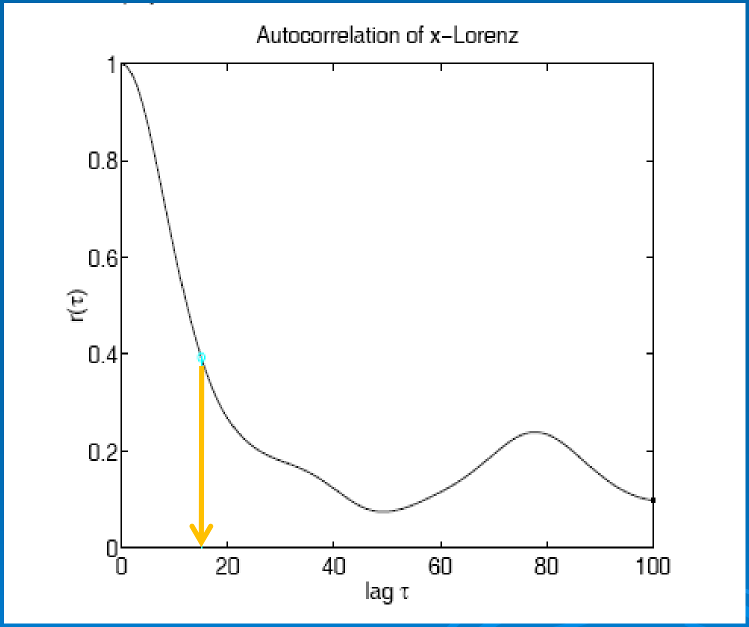
- An empirical rule of thumb that has been established for the chooice of τ is to choose the smallest τ value for which the components of the reconstructed state vector $\underline{x}_i = \left[x_i, x_{i-\tau}, x_{i-2\tau}, \ldots, x_{i-(m-1)\tau}\right]$ become uncorrelated.

- Two methods are employed
  - autocorrelation function
  - average mutual information

Choice of time delay: Autocorrelation function (AF)

- We choose the time τ for which AF is zero for the first time.
- If AF does not fall to zero quite fast we chose as τ the time for which it falls to 1/e (approximately 40%) of the value for τ=0.



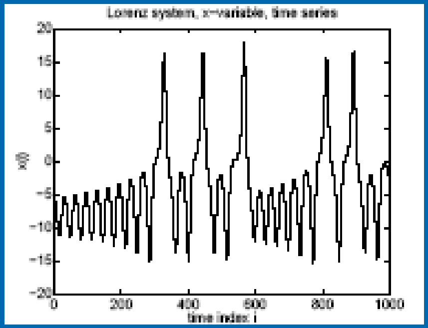


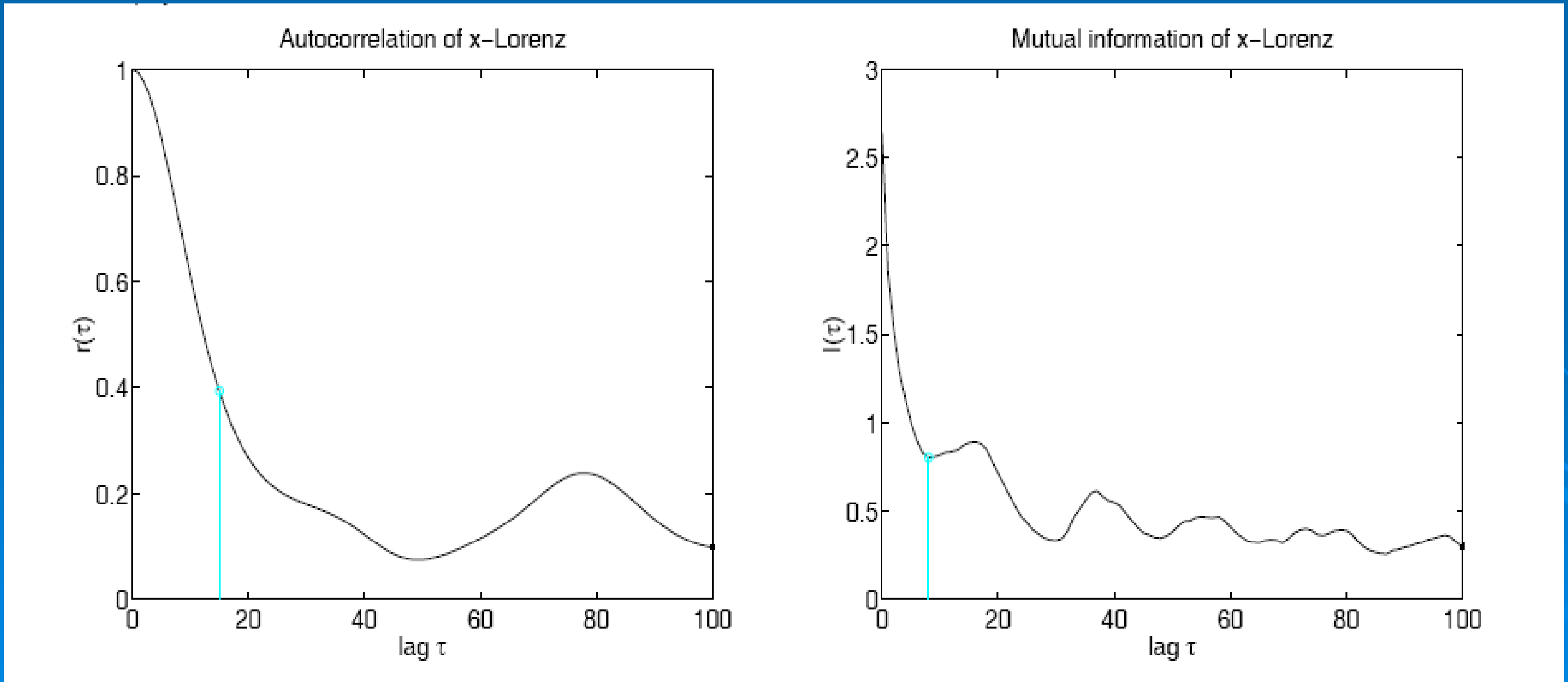
- Remember: AF takes into account only linear correlations between successive points of the time series.

Example of choice of τ

- Lorentz system (only one component)

$$\dot{s}_1 = -a(s_1 - s_2)$$
$$\dot{s}_2 = -s_1 s_3 + b s_1 - s_2$$
$$\dot{s}_3 = s_1 s_2 - c s_3$$
a=10, b=28, c=8/3





Choice of embedding dimension m

Takens' theorem sets the condition $m \geq 2D+1$ where D is the attractor dimension.

Other researchers set a more loose condition: $m \geq D$

If we choose small embedding dimension m the attractor will be "squeezed" and will present self crossing, thus not being equivalent to the original attractor.

If the embedding dimension is larger than what is necessary the corresponding calculations will be unnecessarily more complex and time consuming.

Method of False Nearest Neighbors

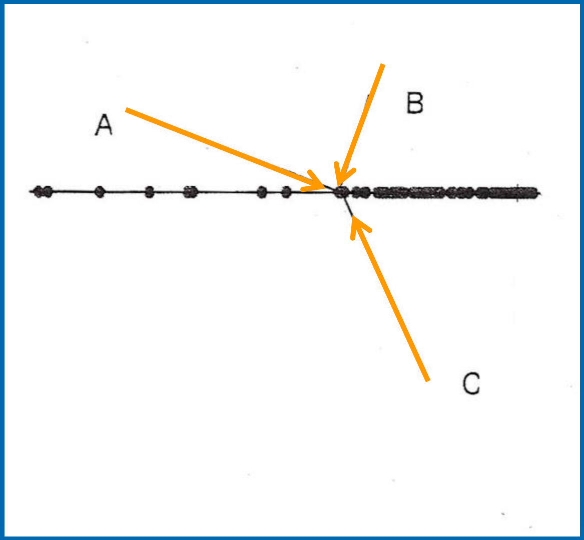
The principle of the method (Abarbanel 1993)

Two points S_i , S_j in the reconstructed phase space with embeding dimension m are located at a distance R. How this distance is affected when the embedding dimension is increased by +1, i.e. becomes m+1 ?

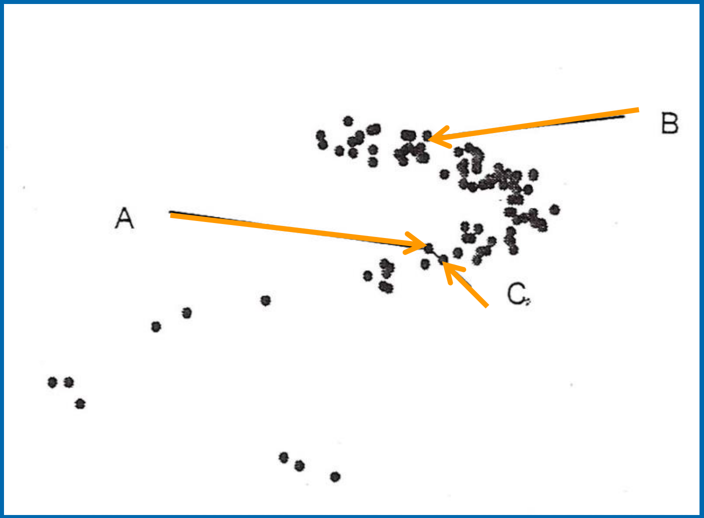
If the distance increases then the attractor can unfold in an additional dimension (and as a result to be better reproduced)
If the distance does not increase we can accept this distance as correct/appropriate.

Method of False Nearest Neighbors

Embedding dimension m



Embedding dimension m+1

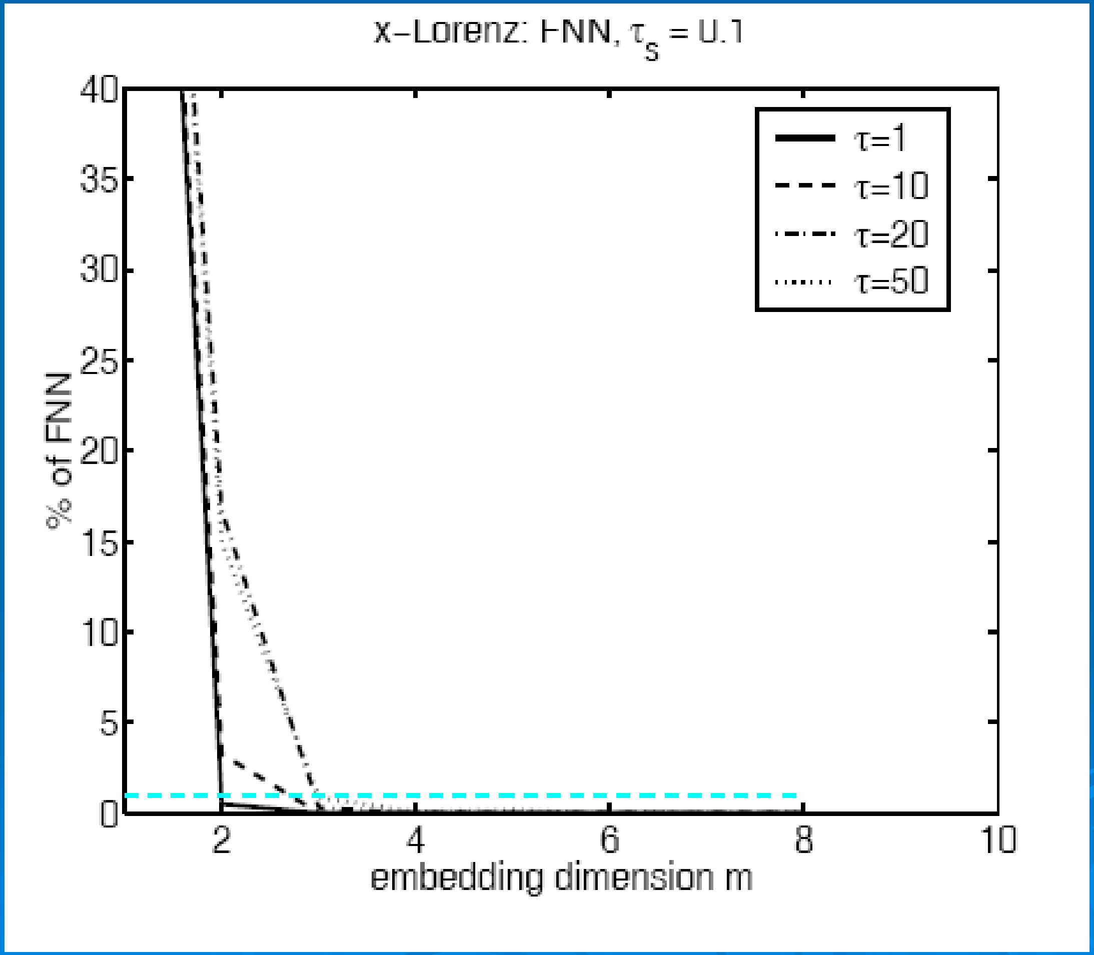


At small embedding dimension the attractor's points are quite close and points A, B and C seem to be neighbors.

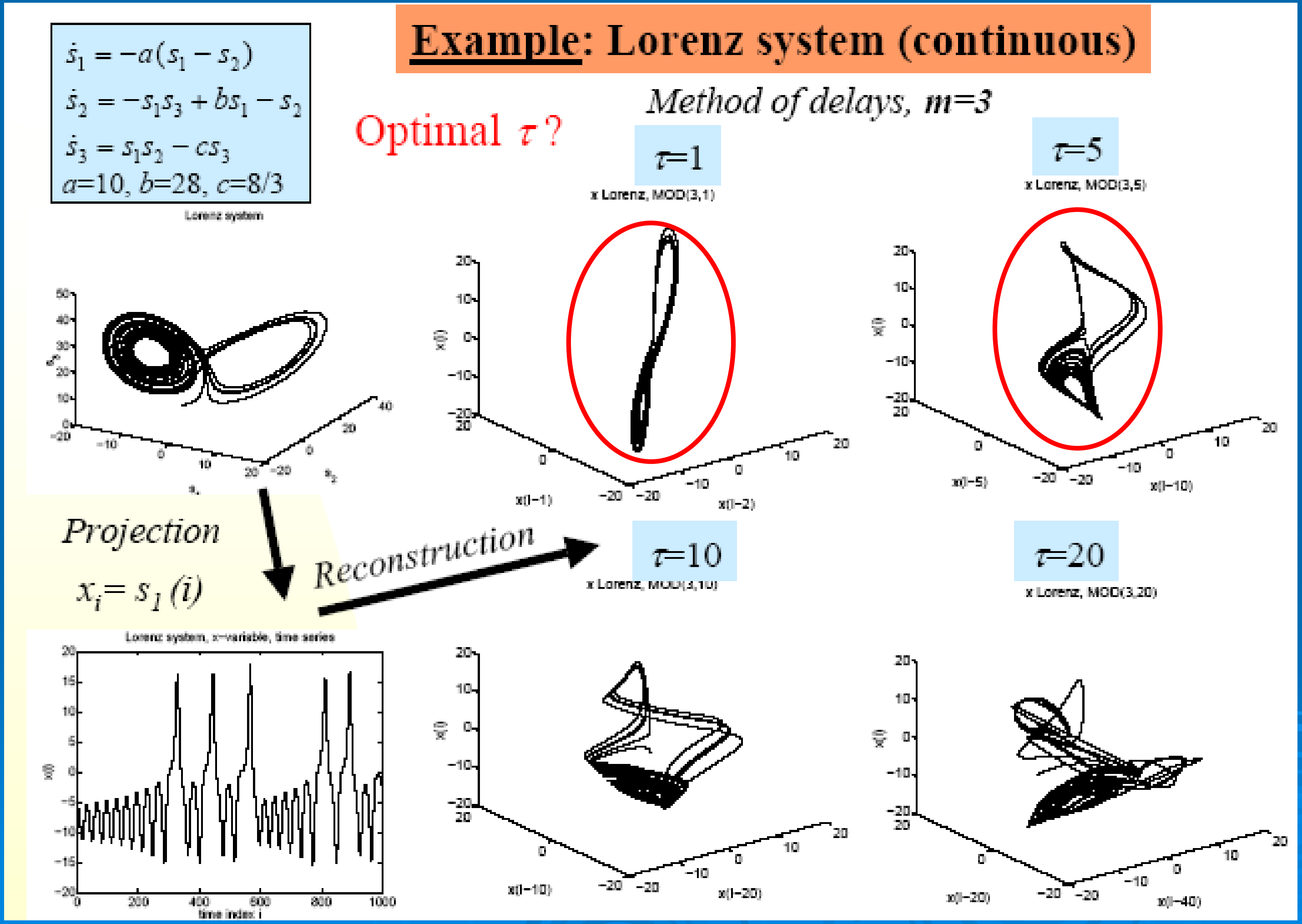
Increasing the embedding dimension by +1 point C remains neighbor of point A However point B moves away from the neighborhood of point A.
Thus point B is a false nearest neighbor of point A. In contrast point C is a true neighbor of point A.

Method of False Nearest Neighbors

Application to the Lorentz system



Choice of reconstruction parameters: time delay and embedding dimension



Kugiumtzis 1999

Phase space reconstruction based method
Recurrence Plots (RP)

- Graphical tool for the qualitative assessment of time series introduced by J.P.Eckmann et al (1987)
- Investigate the phase space trajectory through its graphical representation
- Based on the reconstruction of phase space
- Recurrent points: points close in the phase space

Constructing Recurrence Plots

- step 1: Phase Space Reconstruction
- step 2: Calculate the distances between the state vectors
- step 3: Set a cut off value ε for the distance

which are within a distance ε are defined as recurrent points

$$R_{i,j}^{m,\varepsilon_i} = \Theta(\varepsilon_i - \left\| \vec{x}_i - \vec{x}_j \right\|), \quad x_i \in R^m, \quad i, j = 1 \ldots N$$

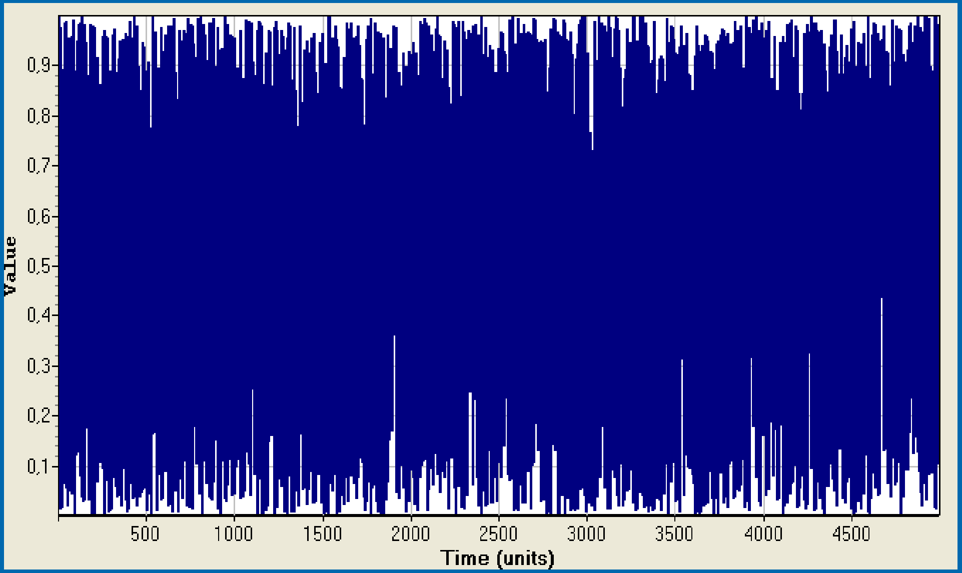
- Colored RPs are showing the same thing (color scaling recurrence points)
- RPs are symmetrical by construction to the main diagonal

Several characteristic structures can appear in a R.P.

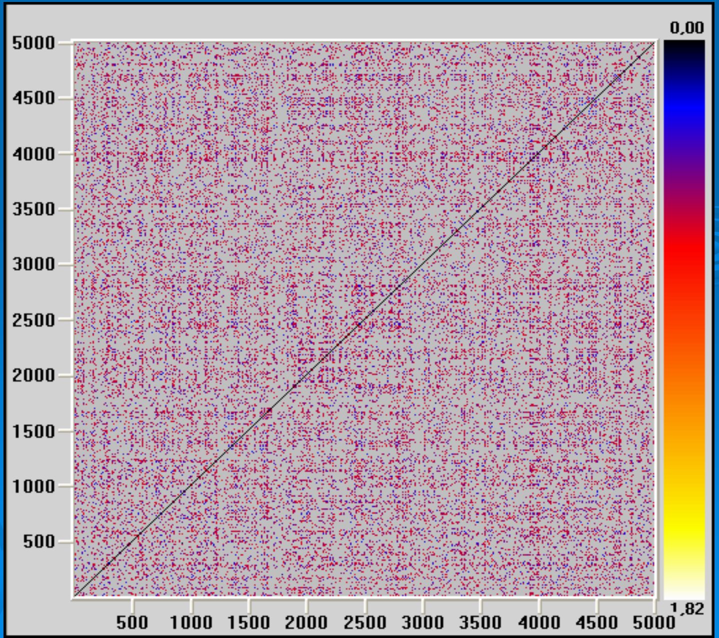
- Single isolated points
  (homogeneity)
- Diagonal lines
  (Trajectory visits the same region of the phase space at different times. Maybe deterministic process)
- Horizontal, Vertical lines/clusters
  (The state is trapped for some time)
- Periodic patterns
  (periodicities)
- White bands
  (abrupt changes in dynamics)

White noise

Time series

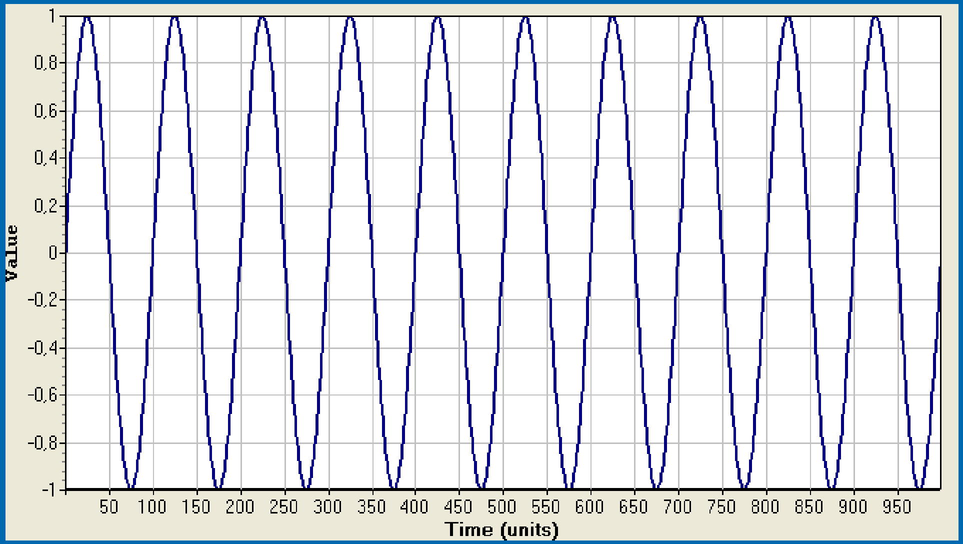


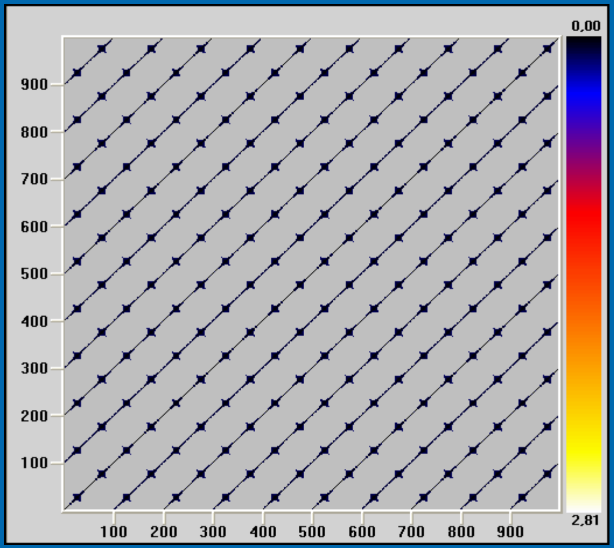
Recurrence Plot



- Single isolated points (homogeneity)
- Neither diagonal nor vertical lines occur

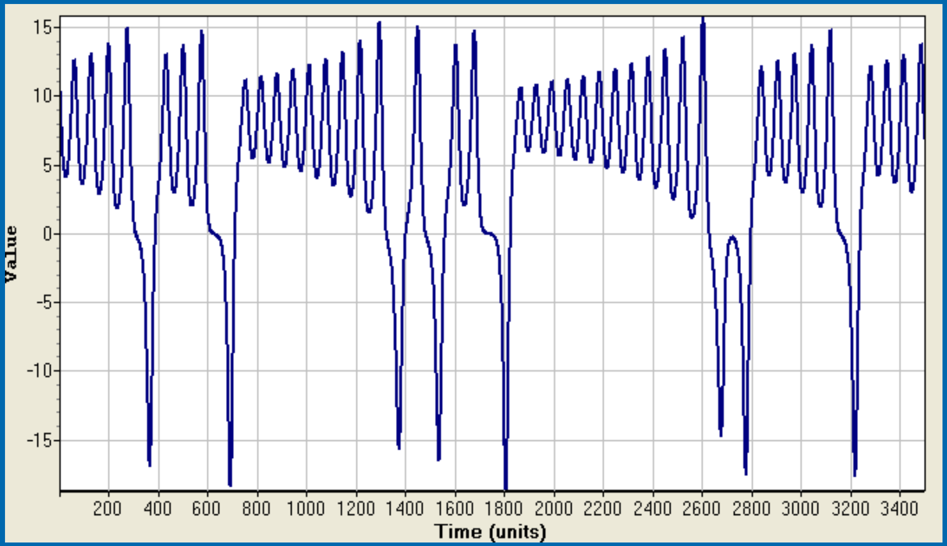
Periodic signal

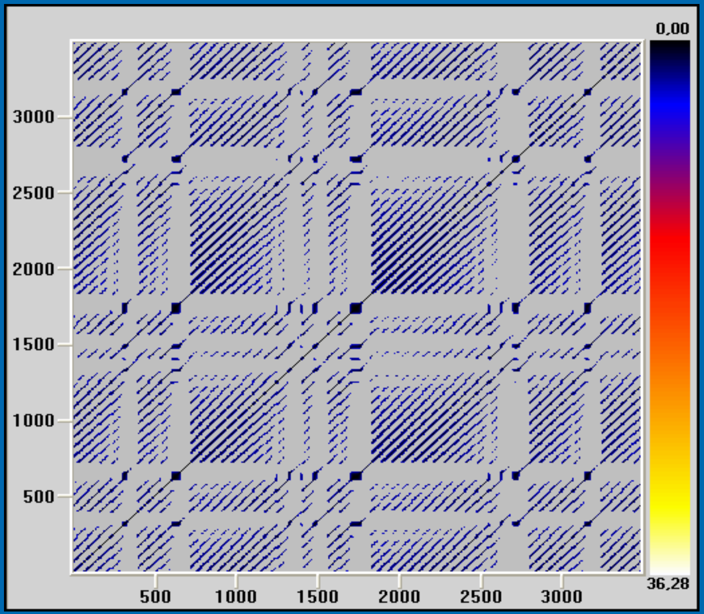




- Structured diagonal lines parallel to the main diagonal line
- No laminar states
- No isolated points
- Strictly periodical structure

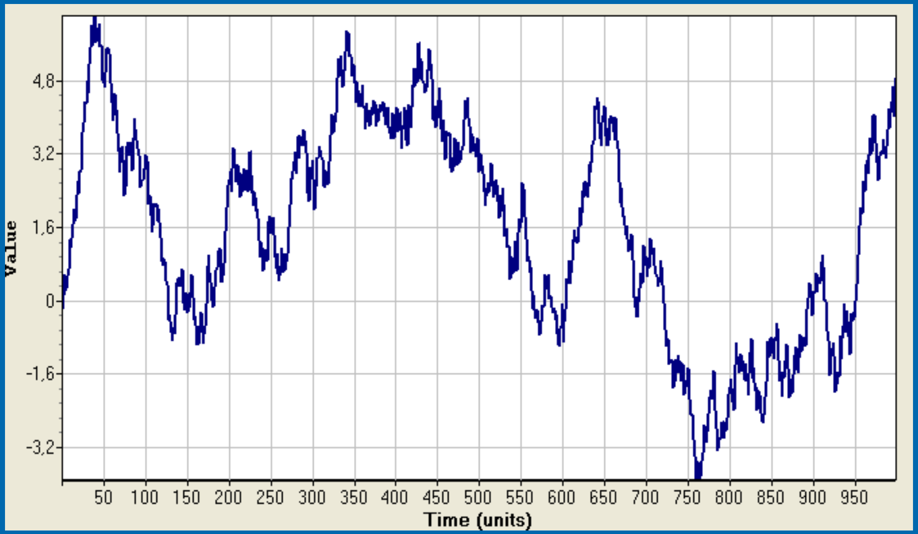
Lorenz (chaotic region)

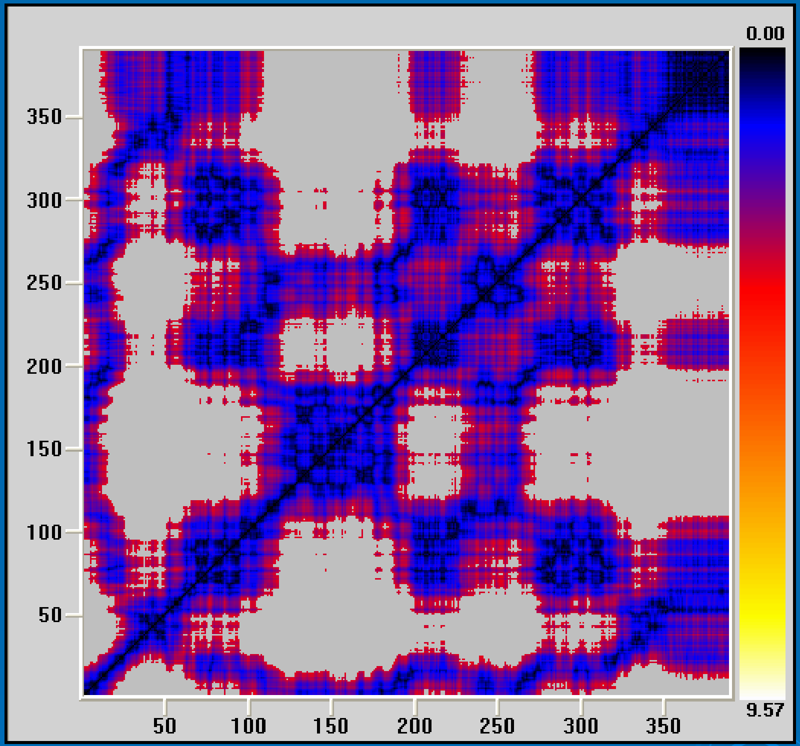




- Parallel lines to the central diagonal (periodicities, deterministic structure)
- We observe increased laminarity (laminar states)

Brownian motion





- Lines Parallel to the main diagonal (deterministic structures)
- White areas (abrupt changes in dynamics.)

Recurrence Quantification Analysis

- Zbillut and Weber (1992)
- %recurrence (ratio of the number of recurrence points (pixels) to the total number of points (pixels) of the plot

$$\%REC = \frac{\sum_{i,j=1}^{N} R_{i,j}}{N^2}$$

$$R_{i,j} = \begin{cases} 1 & ,(i,j) \text{ recurrent} \\ 0 & , \text{otherwise} \end{cases}$$

- %determinism (number of recurrent points forming diagonal lines to the whole number of recurrent points)

$$\%DET = \frac{\sum_{l=l_{\min}}^{N} lP(l)}{\sum_{l=1}^{N} lP(l)}$$

- MaxLine (longest diagonal line segment)

$$L_{\max} = \max(\{l_i; i = 1, \ldots, N_l\})$$

- Trapping Time (average length of the vertical/horizontal structures)

$$TT = \frac{\sum_{\upsilon=\upsilon_{\min}}^{N} \upsilon P(\upsilon)}{\sum_{\upsilon=\upsilon_{\min}}^{N} P(\upsilon)}$$

RQA outputs

Outputs	White noise (5.000 points)	Sinus (1.000 points)	Lorenz (3.500 points)	Brownian (1.000 points)
%deter	0.12	100	99.7	83.6
TrTime	0	2.5	3.5	3.5
MaxLine	12	897	685	919
Trend	-0.007	-0.029	-0.022	-3.5

- All outputs for white noise tend to zero.
- The Trapping time numbers are small for all cases. Not many states stay trapped in time.
- Comparing to the number of points the time series have, only Sinus and Brownian has long deterministic lines.
- Brownian motion has trend as already known

Applications of Recurrence plots

Case 1

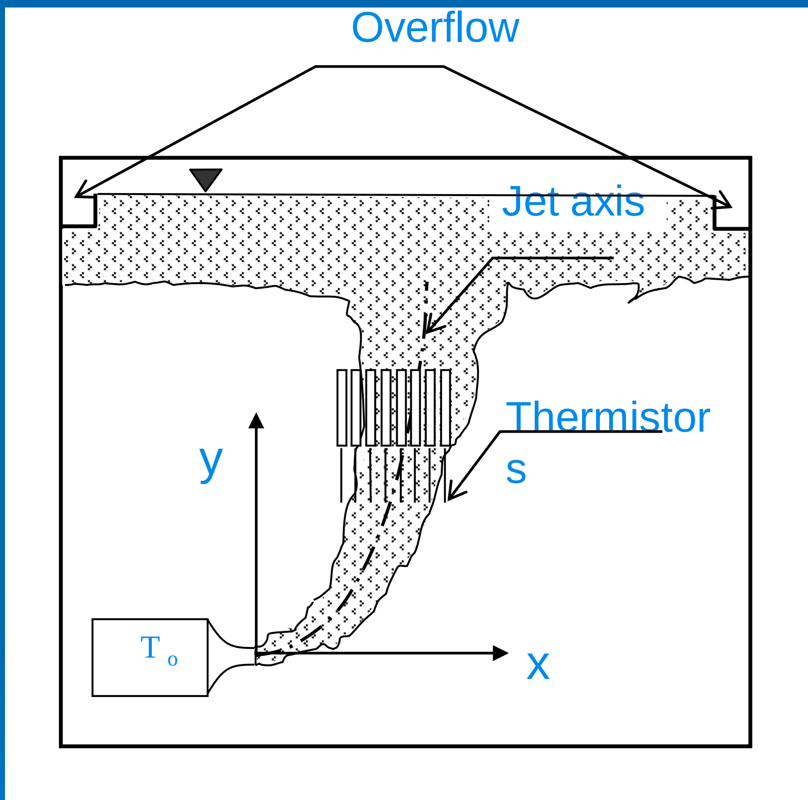
Identify spatial variations (spatiotemporal phenomena)

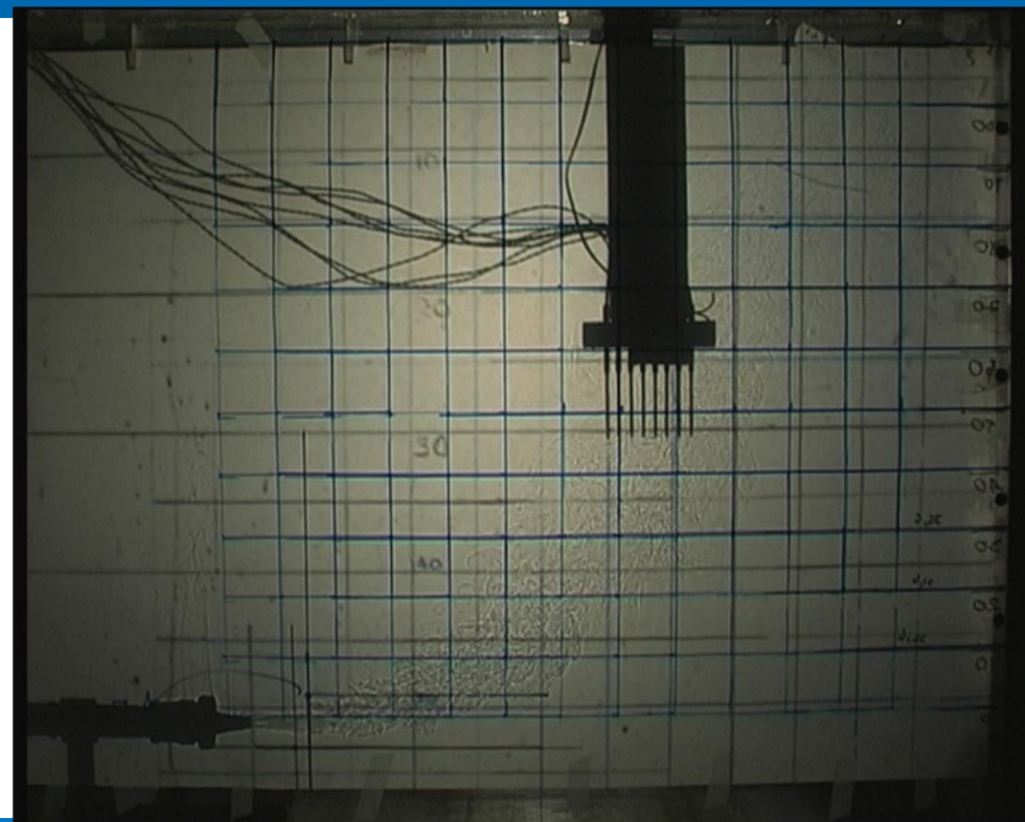
Case 2

Identify transitions in the system evolution

Case 1: Experimental Turbulent heated jet

Karakasidis, et al. (2007). *Physical Review E*, *76*(2), 021120.





Motivation : localization of the jet axis, usually estimated by optical methods. The location of the axis permits to identify the dynamics and perform calculations.

Apart the basic science interest there are also application in engineering such as biological purification stations

Horizontal round heated jet: observables

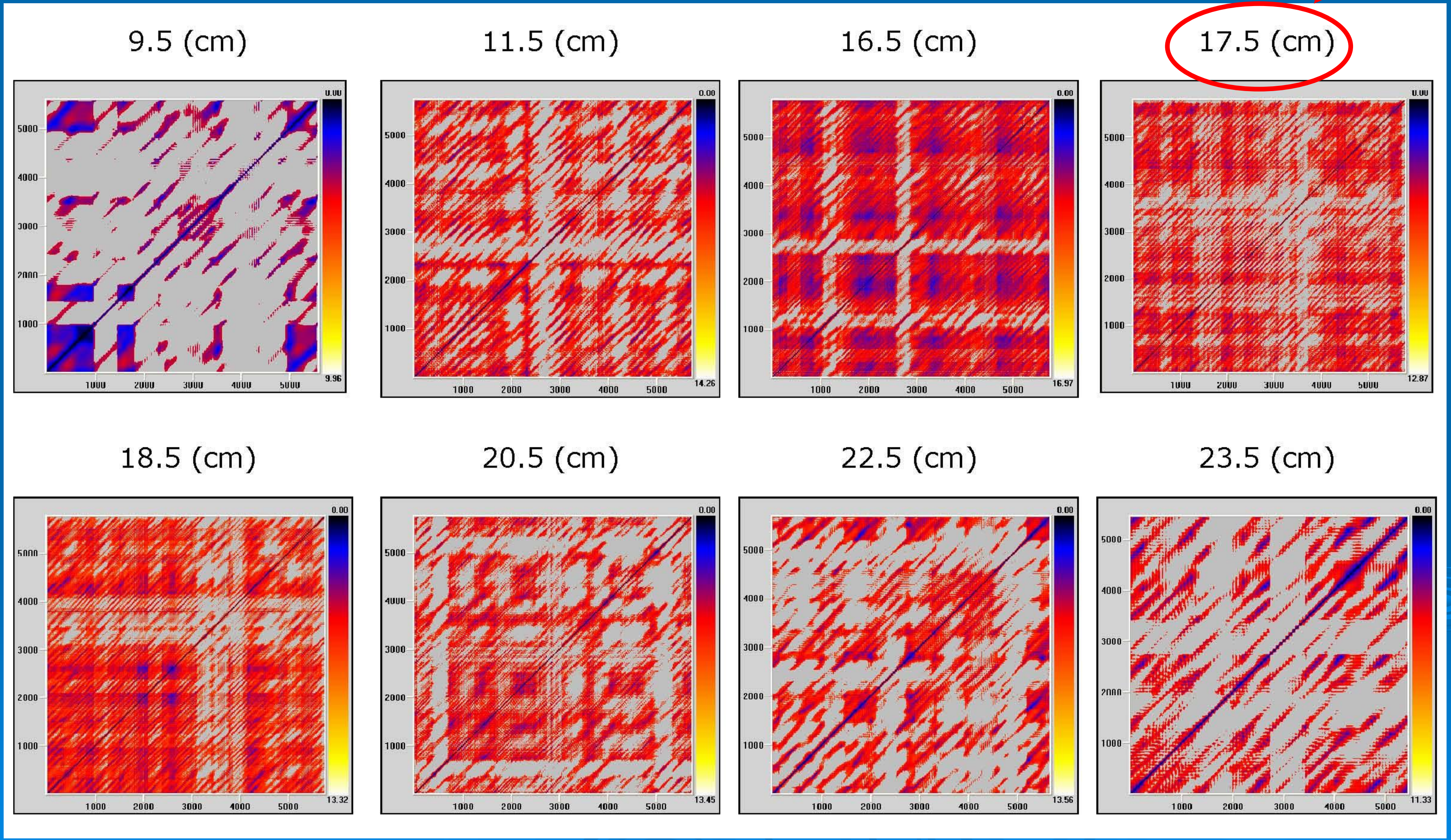
Instantaneous temperature time series were recorded at several points across a horizontal line of the jet as a function of their distance from the center line.

The variation of the characteristic time scales obtained from the above analysis is associated with and interpreted via the transitions of the physical state of the flow from the center line to the boundary of the jet.

Recurrence Plots

Jet axis ?

borders



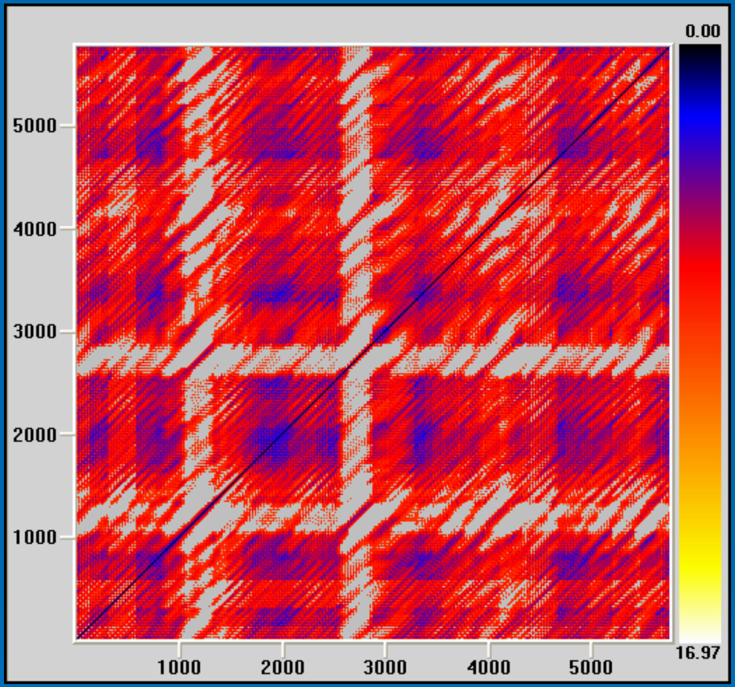
Karakasidis et al 2010

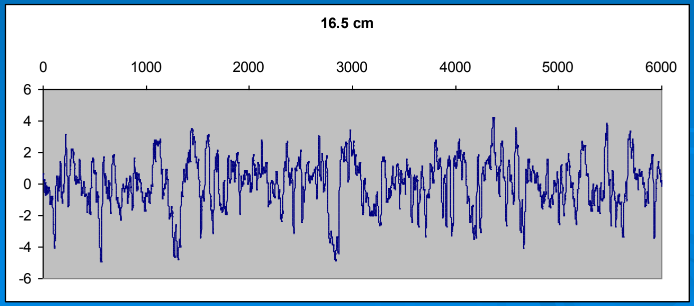
borders

Recurrence Plots again (2)

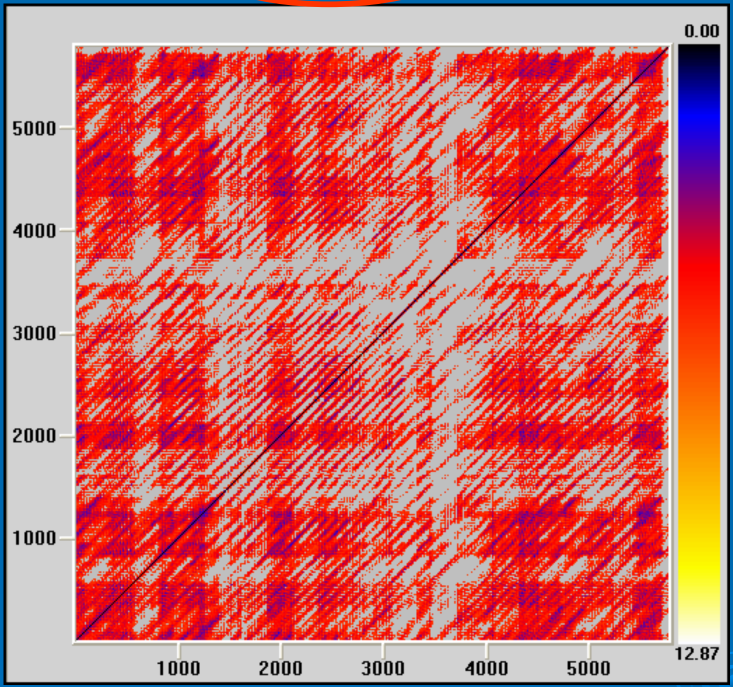
Close to the jet axis

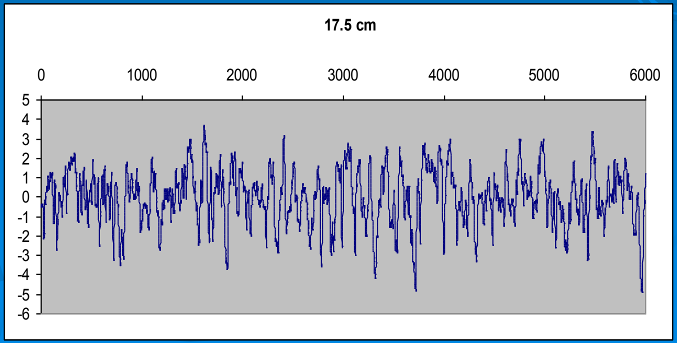
16.5 cm





17.5 cm

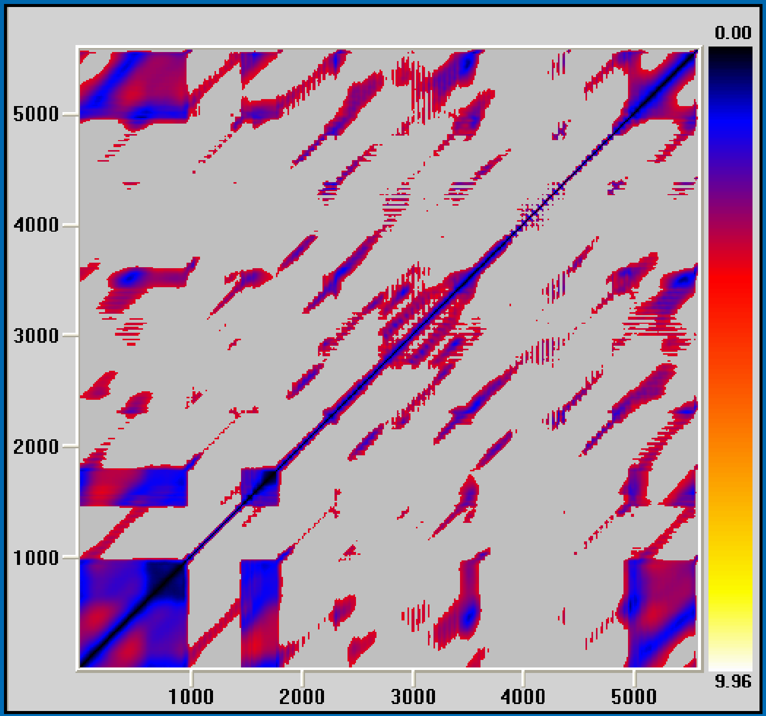


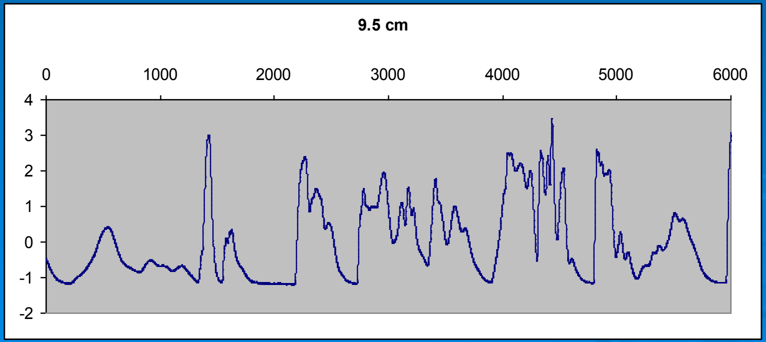


Recurrence Plots again (1)

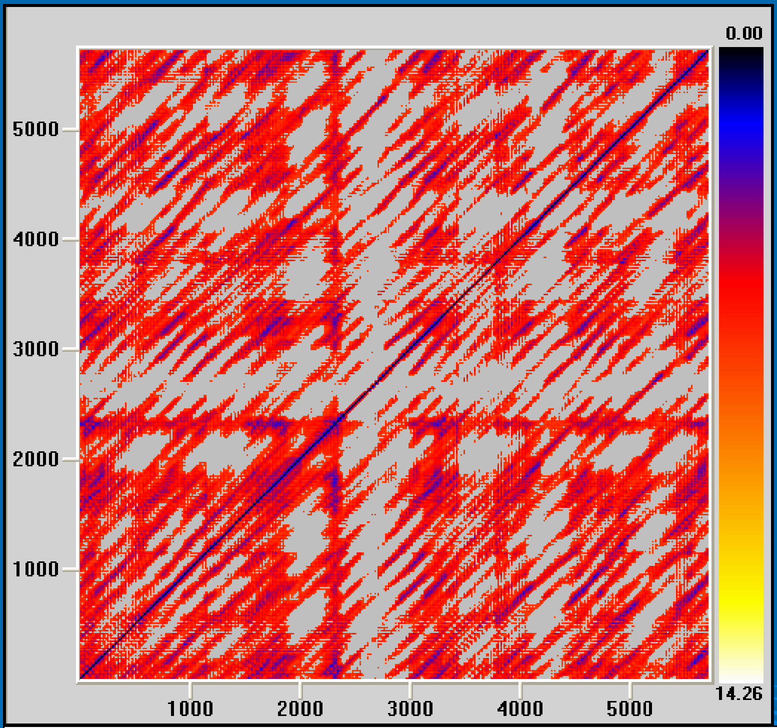
Far from the jet axis

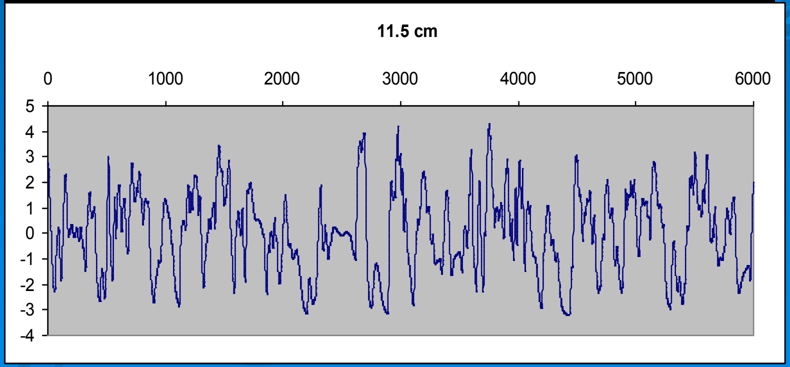
9.5 cm



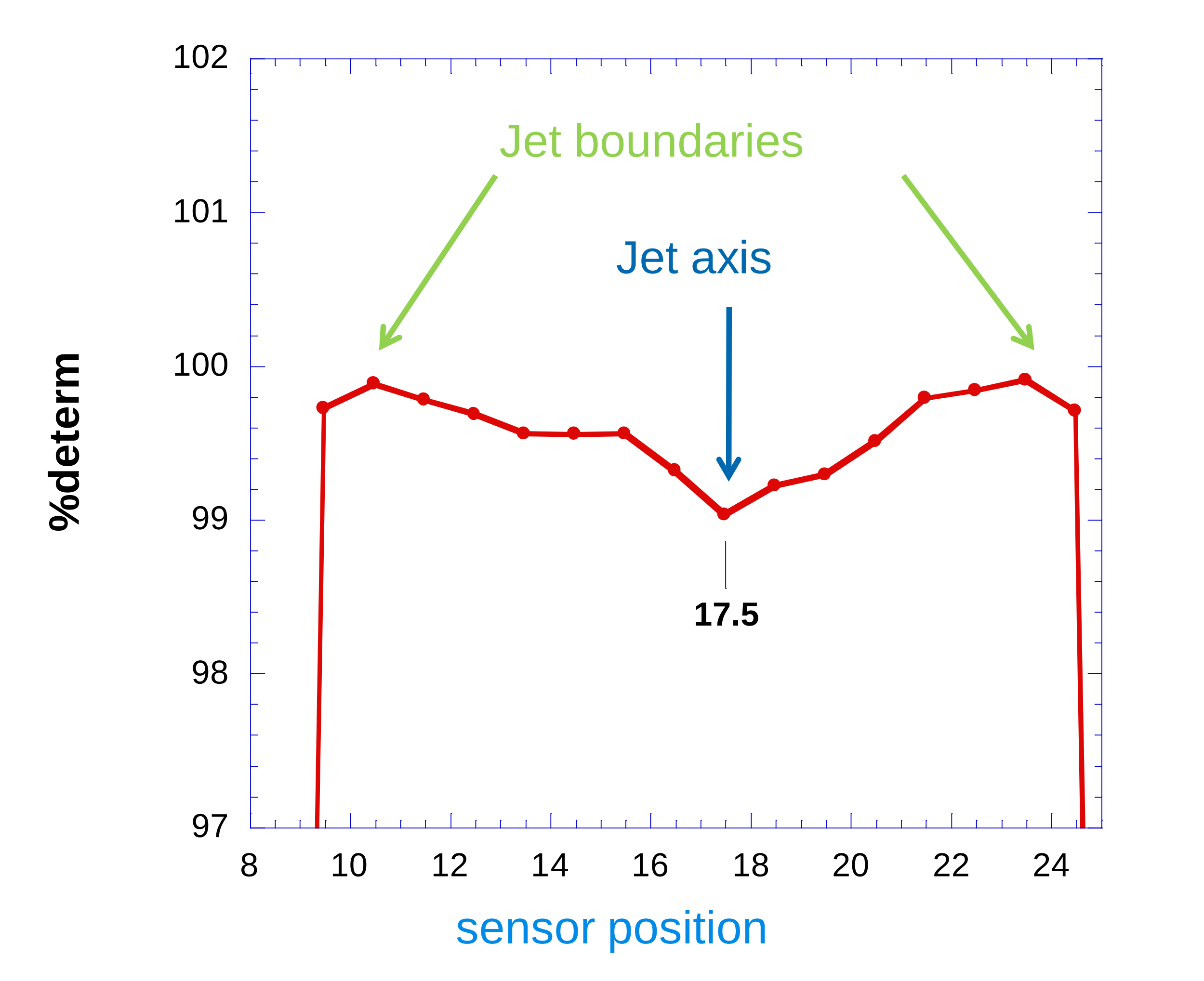


11.5 cm





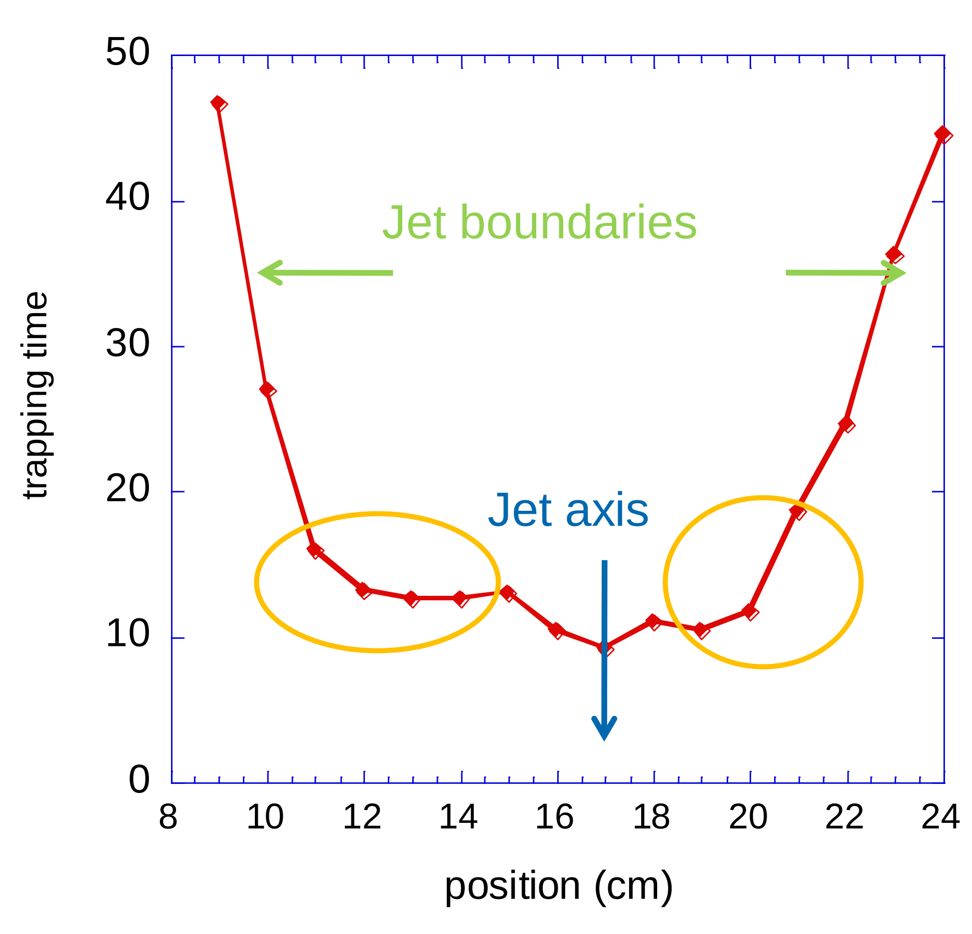
% determ



-Line segments parallel to the main diagonal (high values of %determ)

-Asymmetry relative to the jet axis

Trapping time



Small times correspond to short living flow structures, which could be true in the centerline where we have a region of fully developed turbulence.

As we go towards the ambient fluid, larger scale flow structures dominate the flow.

These flow structures live longer and give rise to longer memory effects.

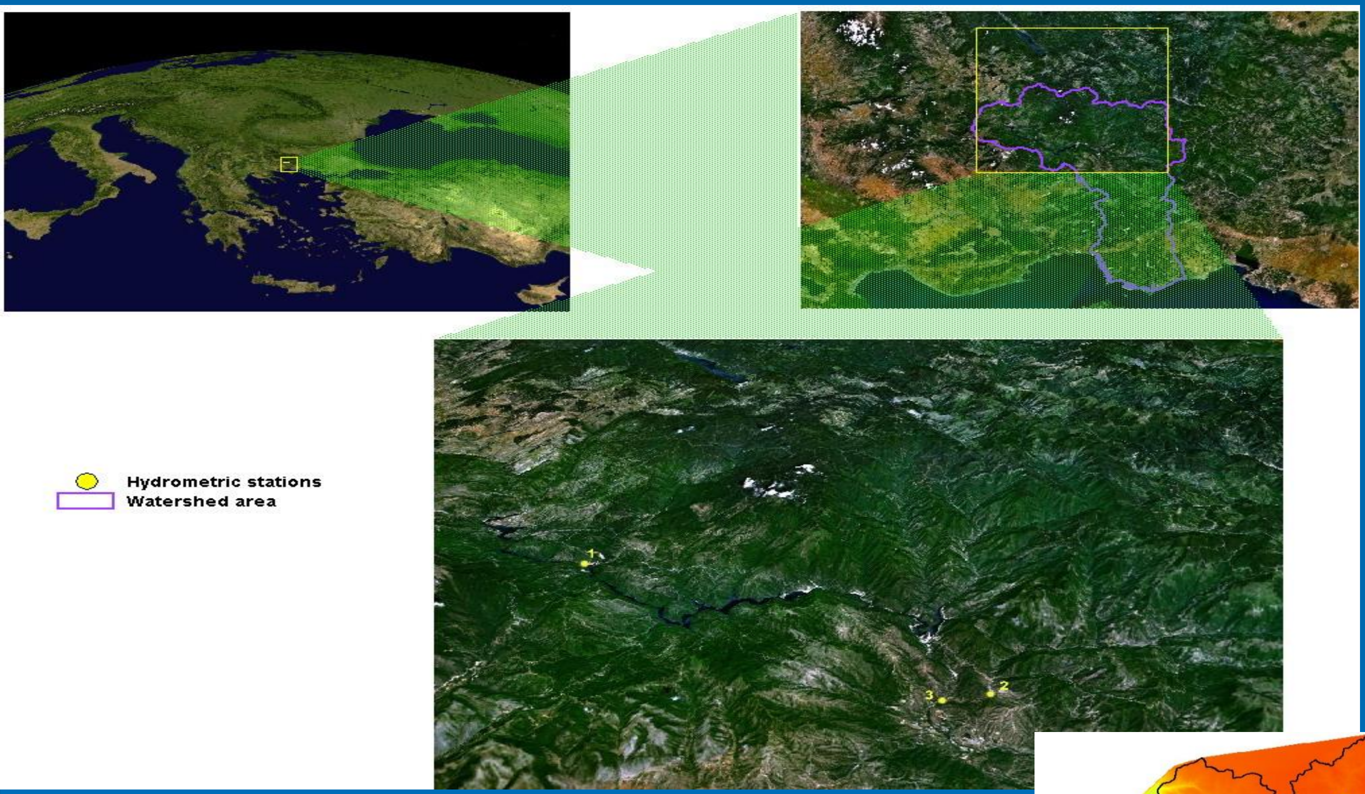
In the ambient fluid trapping times are very small due to thermal fluctuations (no appreciable fluid flow).

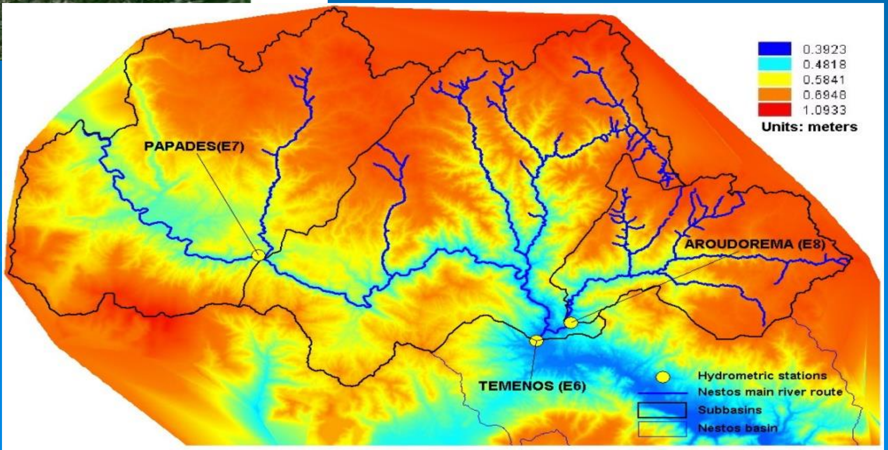
Lowest value at the vicinity of 17.5 cm.

And we have a slight anisotropic behavior.

The lowest value could correspond to the region of fully developed turbulence.

Case 2: Identify variations in the system temporal evolution

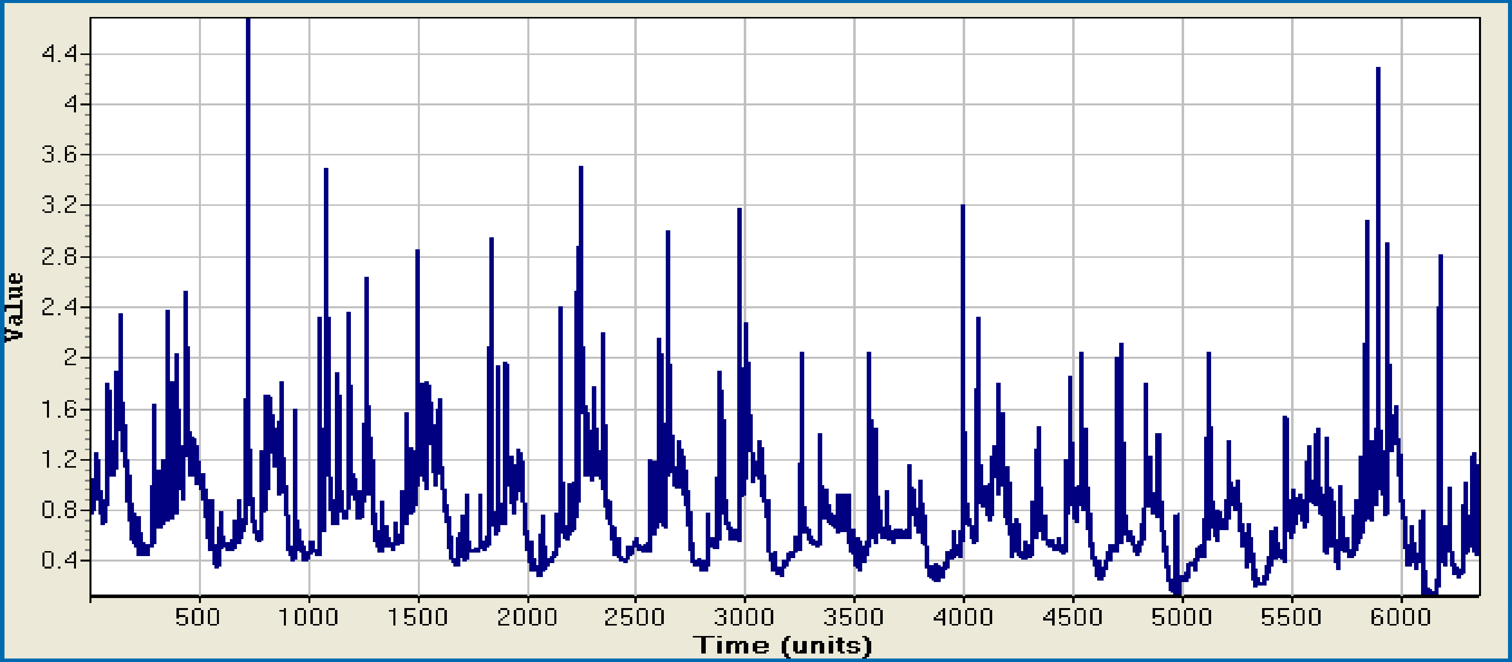




Nestos River Water Level analysis

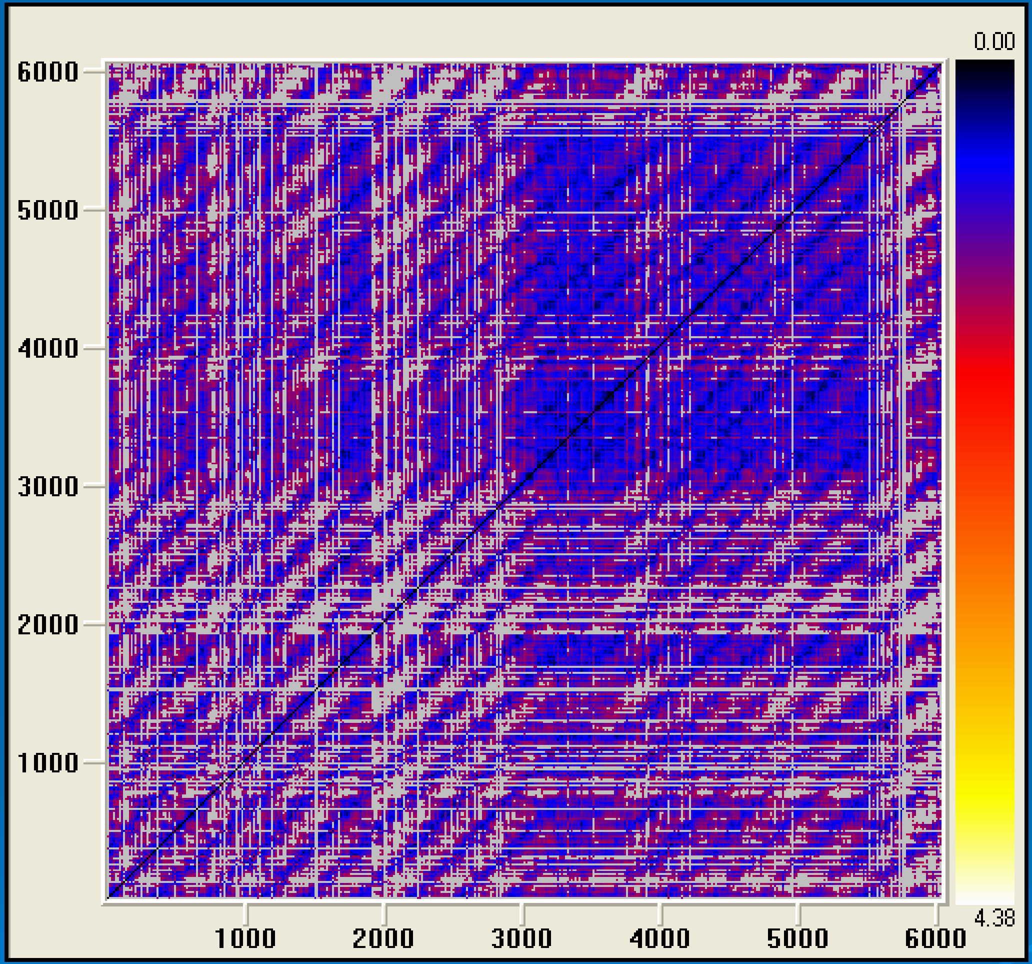
Karakasidis et al 2011

Time series of Nestos River



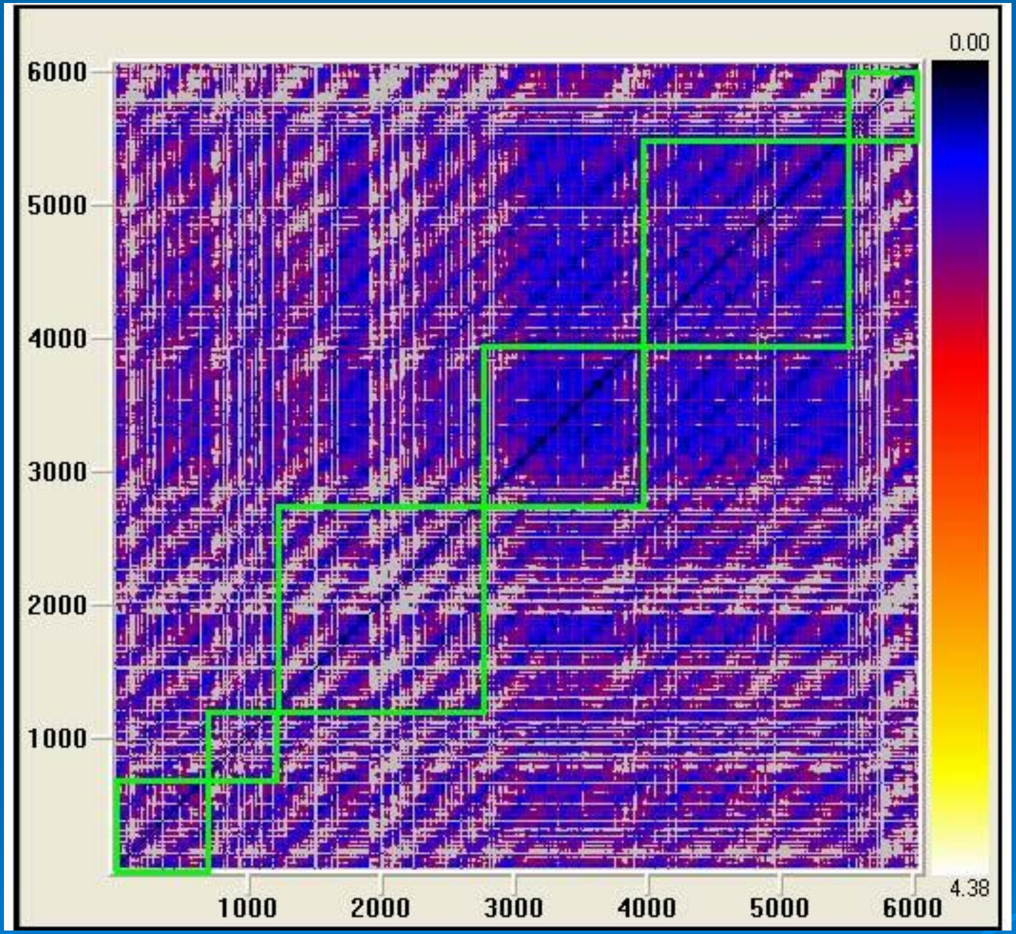
16 years of daily measurements
Periodic view with trend

Global inspection of the RP



- 16 lines parallel to the main diagonal (periodicity)
- Deep blue Recurrence points (close to each other), red Recurrence points (more distant to each other)
- Airplane Structure of the RP (sign of trend in our time series)

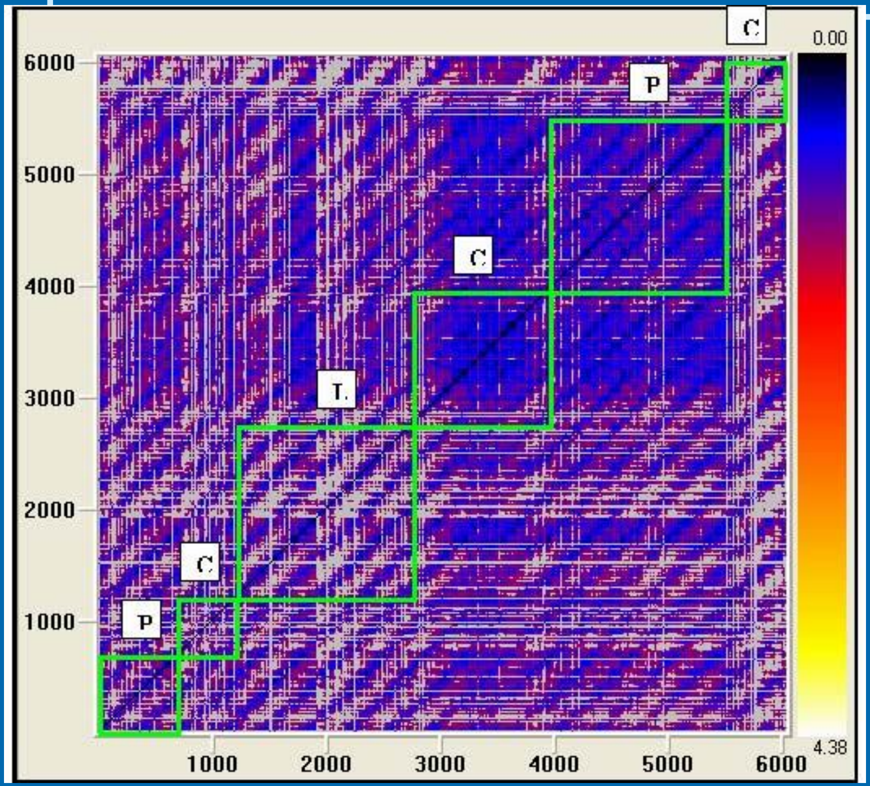
Interesting areas

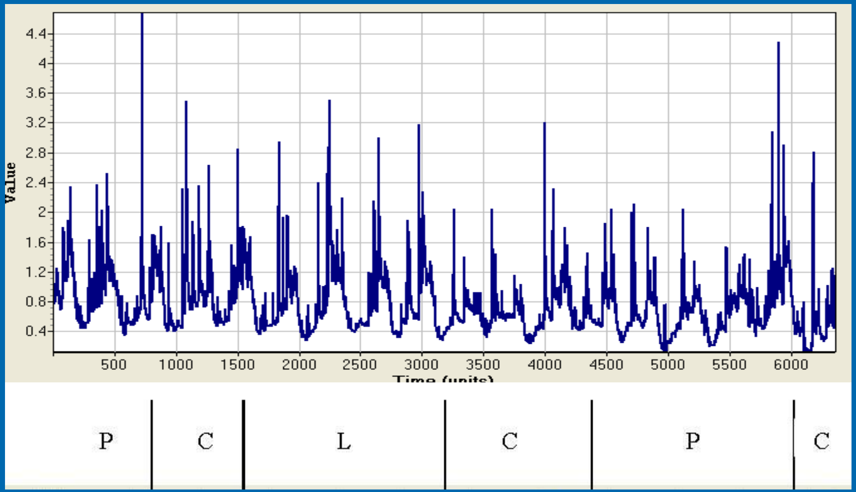


Six interesting areas (system transitions)

(0-650), (650-1300), (1300-2750),(2750-3900), (3900-5500), (5450-6000)

RP of Nestos River water level





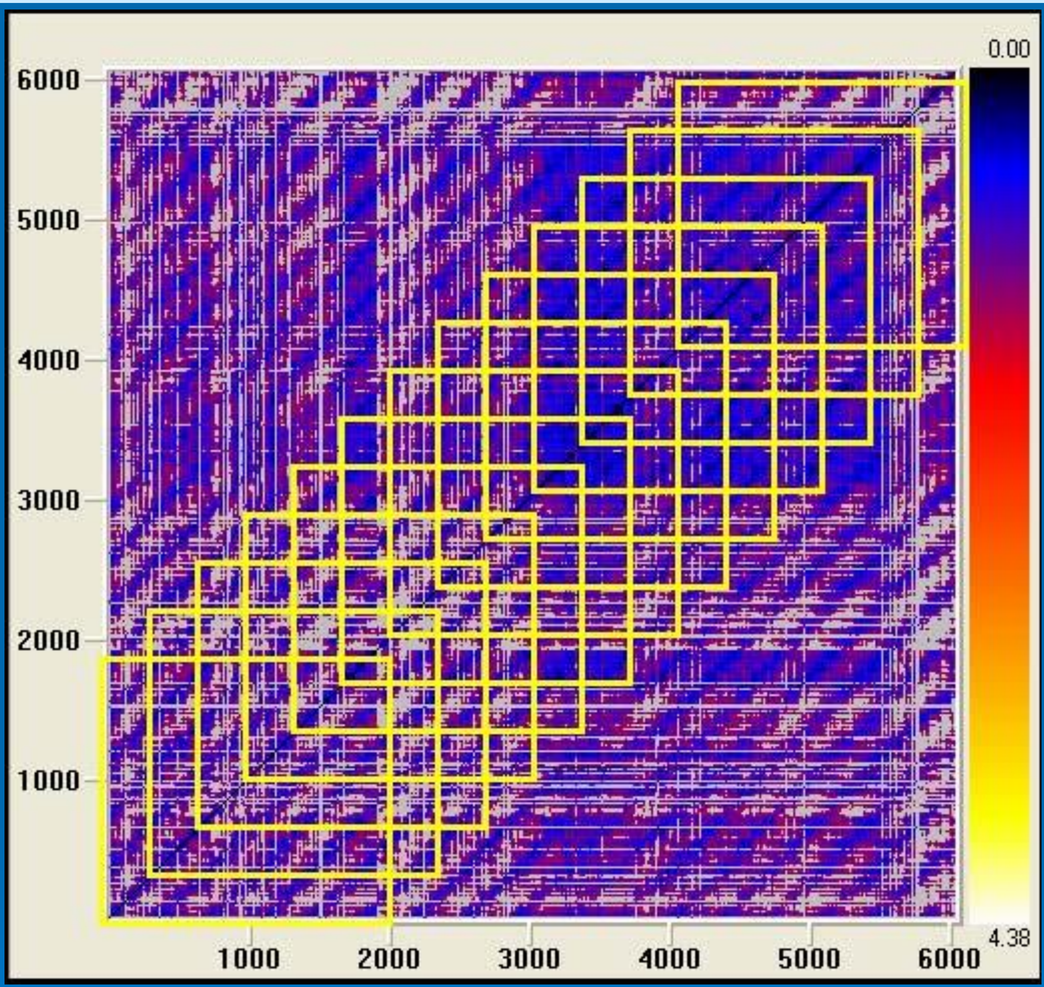
-Global inspection: 16 lines parallel to the main diagonal are observed (periodicity) so as the Airplane Structure of the RP (sign of trend in our time series)

-Closer inspection: "Chaotic" regions (650-1300), (2750-3900), (5450-6000): deterministic lines are very small .

Periodic areas (0-650), (3900-5450):parallel lines to the main diagonal

"Laminar " area (1300-2750): no large deterministic lines (parallel to the main diagonal)

Epoch Analysis



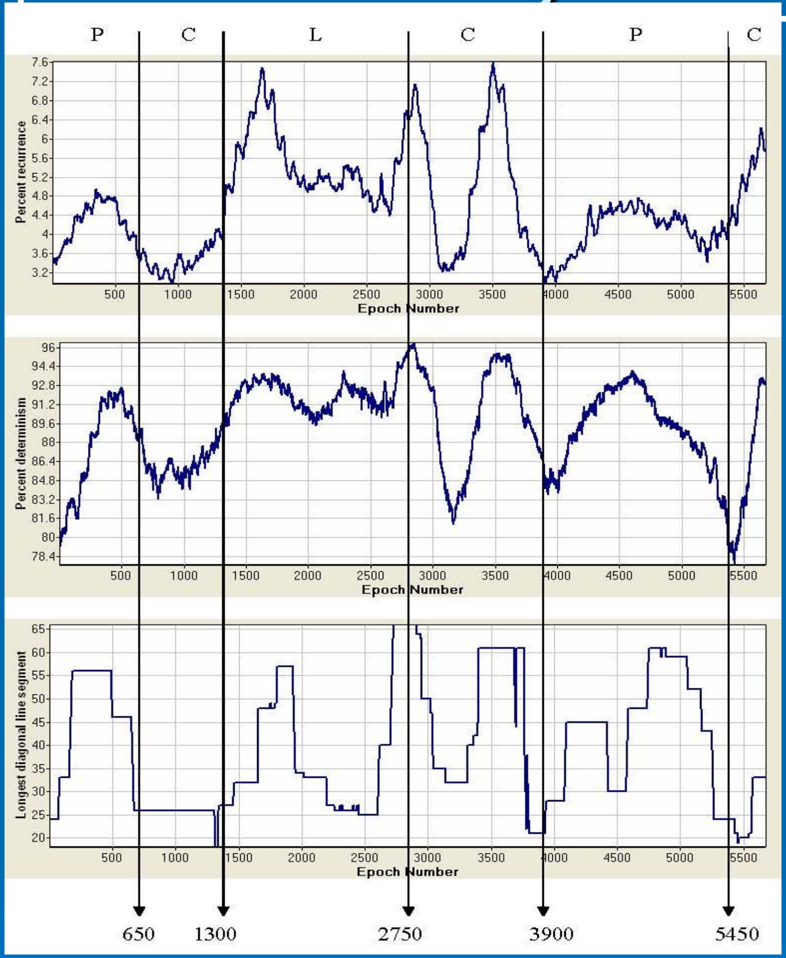
- Quantitative results over sliding windows over the main diagonal (example picture)
- Our Problem

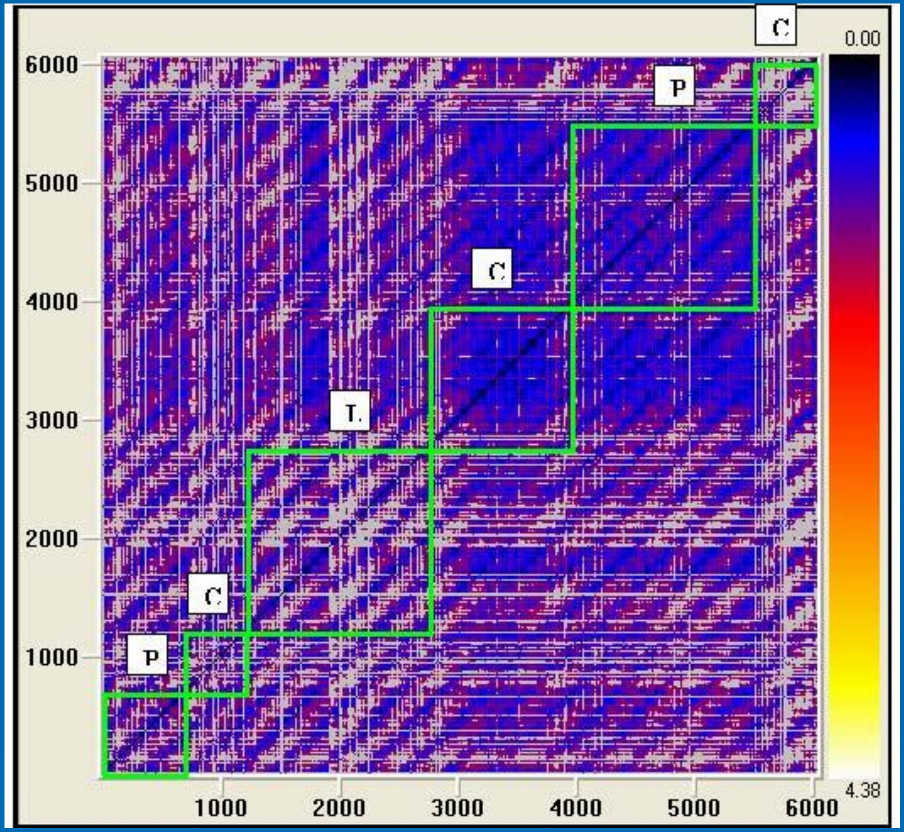
  -Window size: 365 points (1 year)

  -Window shift: 1 point (day)

  -Number of epochs: 5680

System transitions (1)



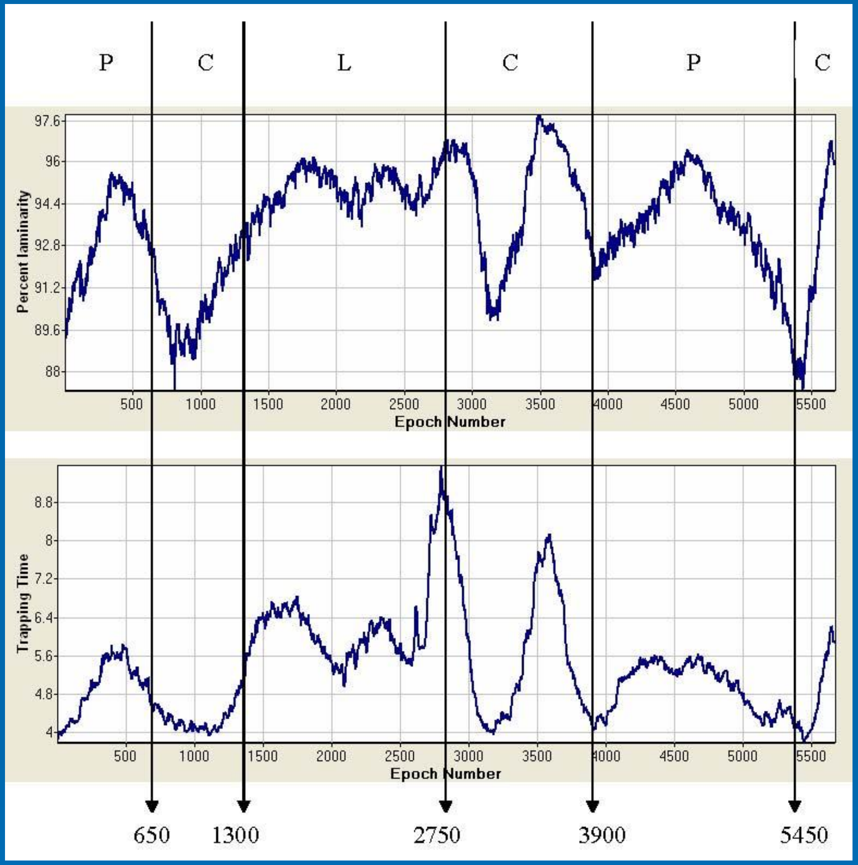


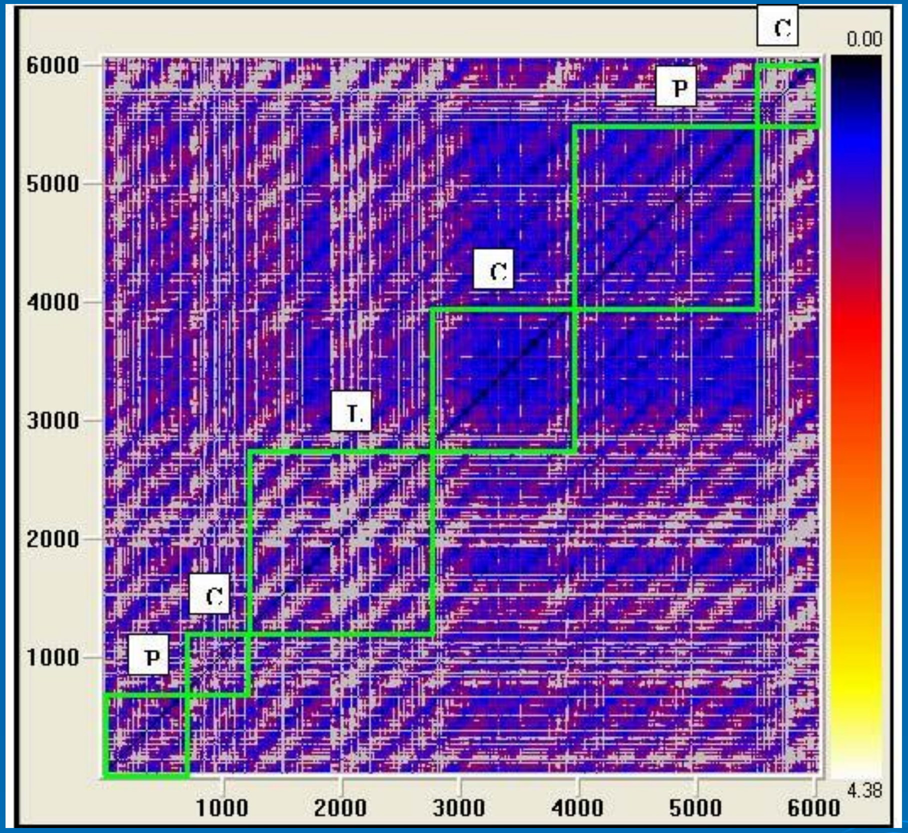
•Pick to pick (small picks) on %recurrence is 84 points, depicts the seasonal changes

Indications of chaos (650-1300), (2750-3900), (5450-6000):low deterministic lines and determinism, abrupt changes of graphs.

Periodic areas (0-650), (3900-5450): high values of %determinism and maximum line (deterministic lines)

System transitions (2)





•State Trapped in time, laminarities (1300-2750)

Laminar state

- Looks like periodic, but it is not: high %determinism, not high deterministic lines parallel to the main diagonal (low maxLine)
- It is "trapped" in time: High Tapping Time

Conclusions

- Nonlinear methods seem to identify system transitions in time or in time and space
- Indications of chaos in the systems studied
- System dynamics are reflected on the R.P.
- Using RQA we can extract useful information about the dynamics of a system
- The methodology seems promising for the study of spatiotemporal and multiscale phenomena in engineering and environmental sciences

“Analysis of turbulent heated jets using non - linear methods and complex network time series mapping”

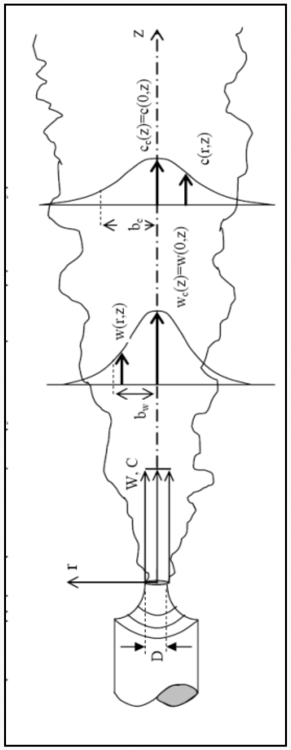
- *Lab. Hydromechanics and Environmental Engineering University of Thessaly*
  - **A. Charakopoulos (Research associate)**
  - **A. Fragkou (Ph.D. student)**
  - **A. Liakopoulos (Professor)**
- *Applied Hydraulics Laboratory of the National Technical University of Athens*
  - **P. Papanicolaou (Professor)**

Charakopoulos, A. K., Karakasidis, T. E., Papanicolaou, P. N., & Liakopoulos, A. (2014). The application of complex network time series analysis in turbulent heated jets. *Chaos*, *24*(2), 024408.

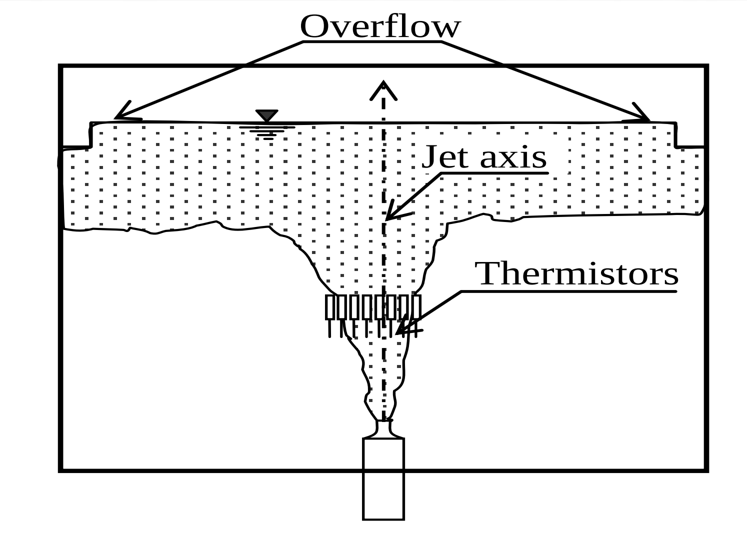
Charakopoulos, A. K., Karakasidis, T. E., Papanicolaou, P. N., & Liakopoulos, A. (2014). Nonlinear time series analysis and clustering for jet axis identification in vertical turbulent heated jets. *Physical Review E*, *89*(3), 032913.

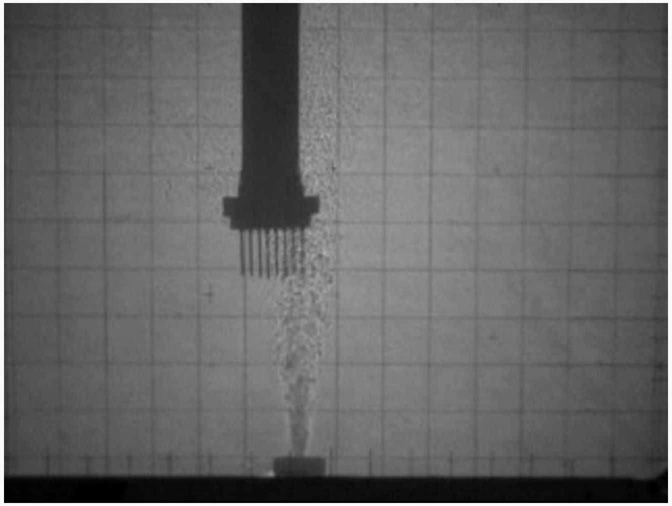
Turbulence Fluid Complexity

Large Number of Environmental and Engineering Application of Turbulent jet flow



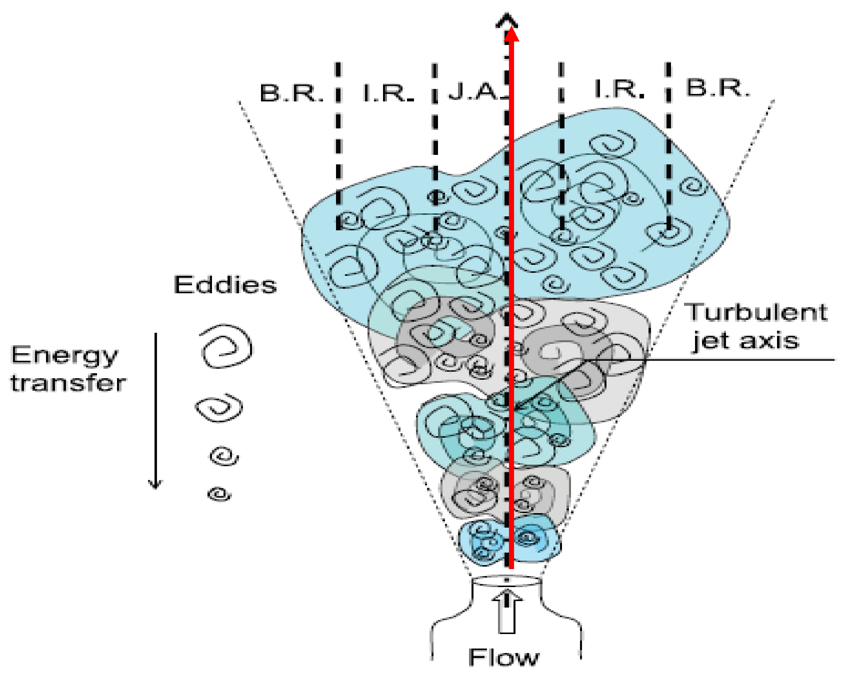
Experimental set up





- Data from different sets of experiments with various initial conditions and circular and elliptical shaped nozzles
- Ambient water temperature ranged between 18.4-24.6 °C while the jet water temperature ranged between 58.6 to 61.4 °C
- Temperature data sampling frequency at 80Hz and 100Hz

Vertical heated jet



- **Three region behavior**
  - The first region corresponds to large distances from the jet axis, actually at the boundary with ambient water named boundary region (BR)
  - The second one, the inner region (IR), concerns the region between the boundary region and the core of the jet,
  - The third region, the jet axis region (JR), is the region near the core of the jet

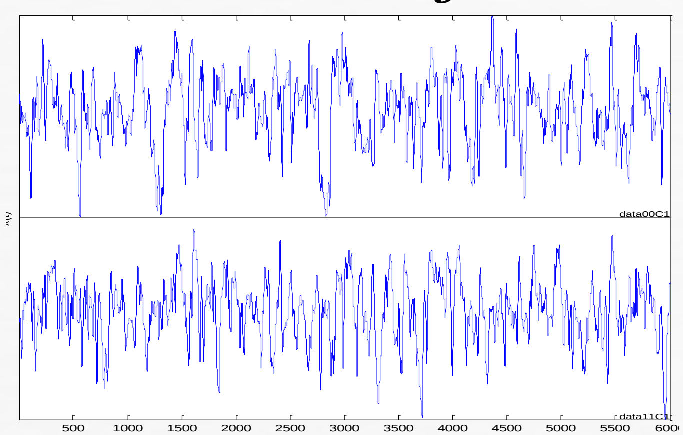
The dynamics of these regions are characterized by the presence of small and large scale structures (vortices)

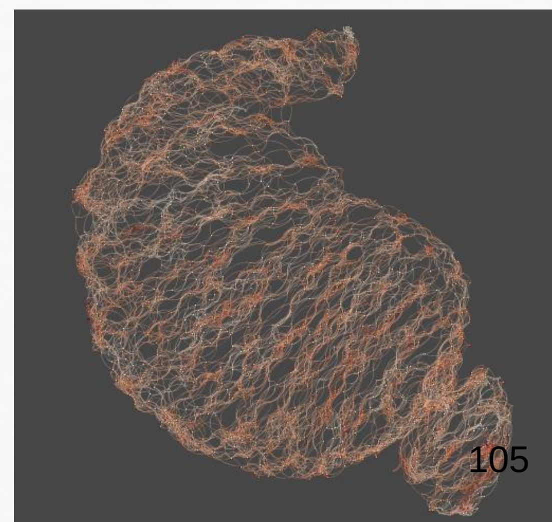
Methodology

Distinguish the state of the fluid region based on time series analysis

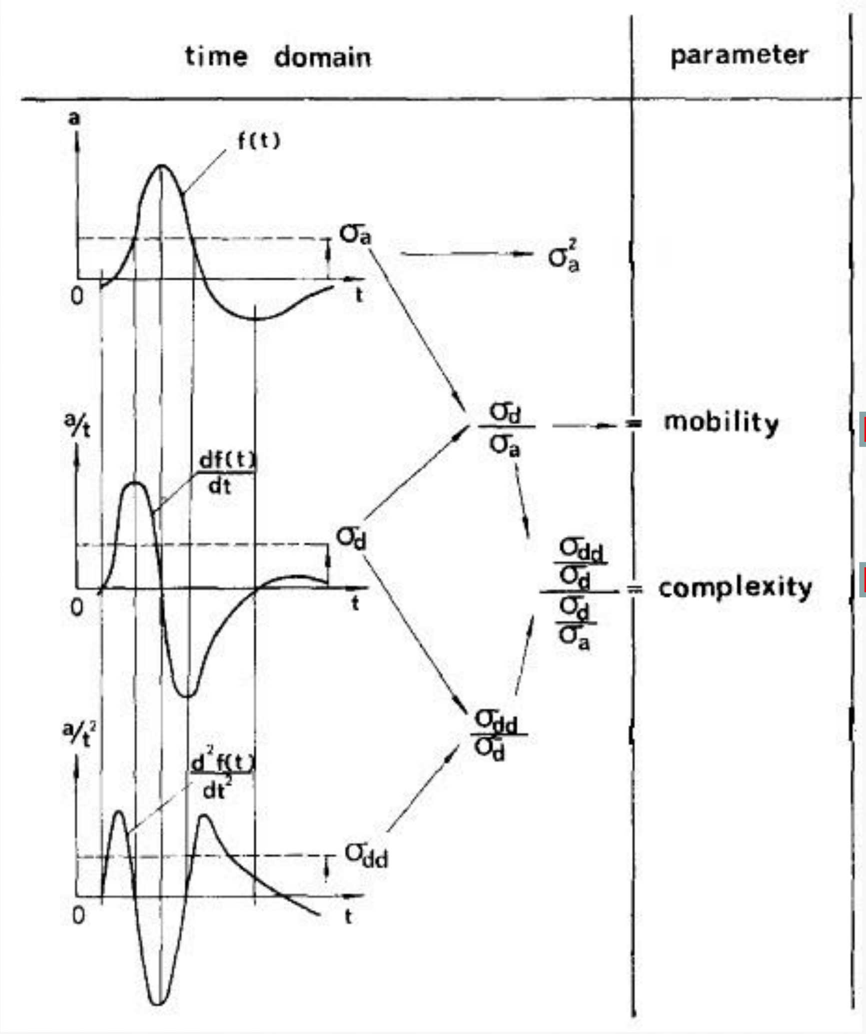
Linear and Non Linear Methods for time series analysis

Complex Network Analysis





Hjorth Mobility and Complexity



Hjorth [1970]

is showing how fast the variance varies in a time series

is the mobility of the Mobility; captures changes in the stages of a time series

Use of measures to perform clustering

Why clustering?

Different measures take into account better dynamical features of the system.

Clustering Method:

Single Linkage Hierarchical Agglomerative Clustering.

No assumption is made apriori on the number of the groups.

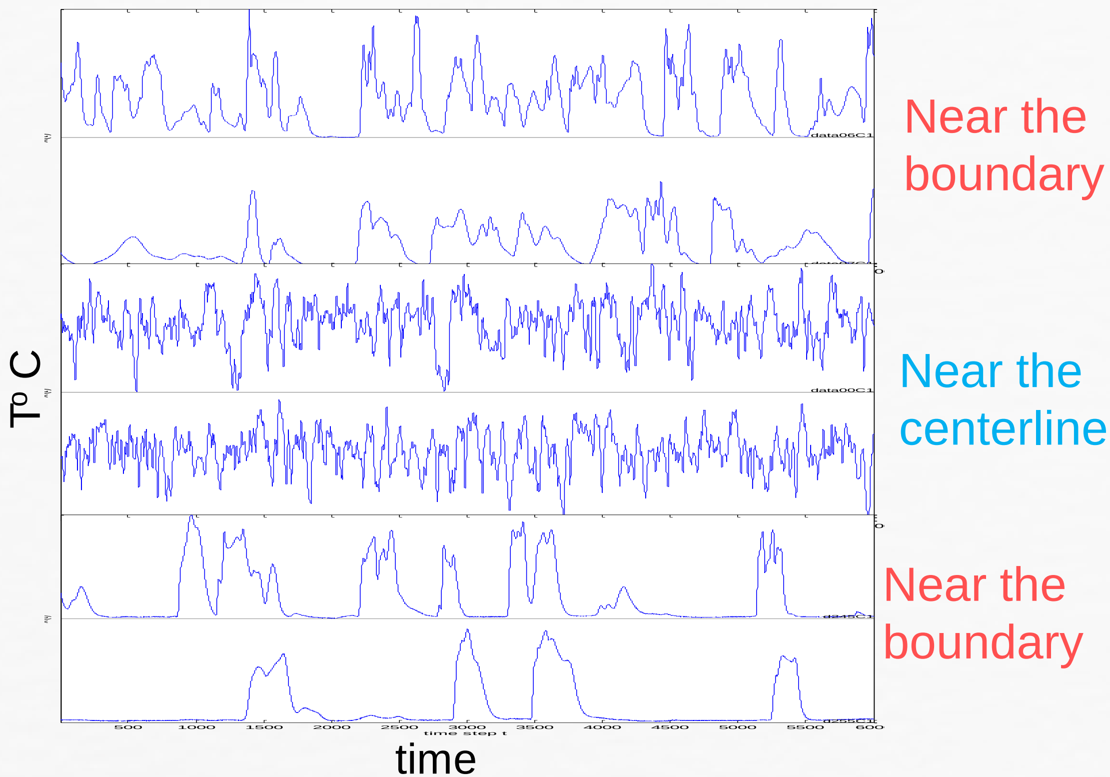
Clustering properties:

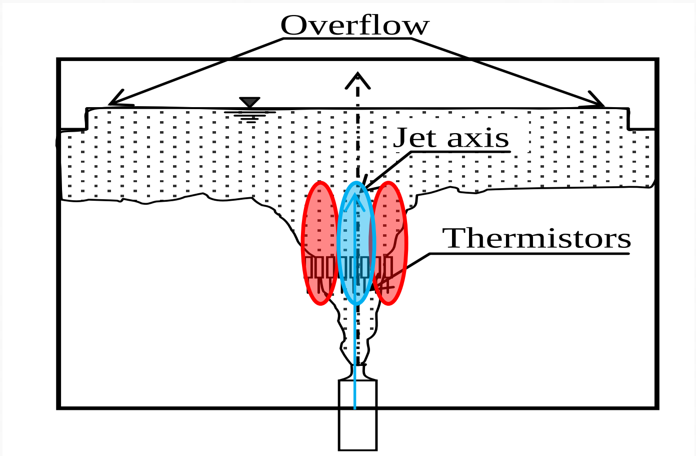
simple statistical measures, Hurst exponent, Hjorth parameters and cumulative mutual information (normalized quantities)

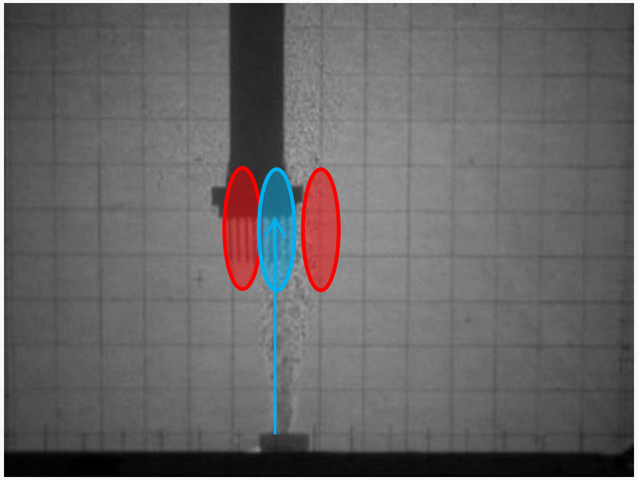
Cases studied

General

✓ *Five set of experimental temperature fluctuations in vertical turbulent heated jets with different initial parameters.*



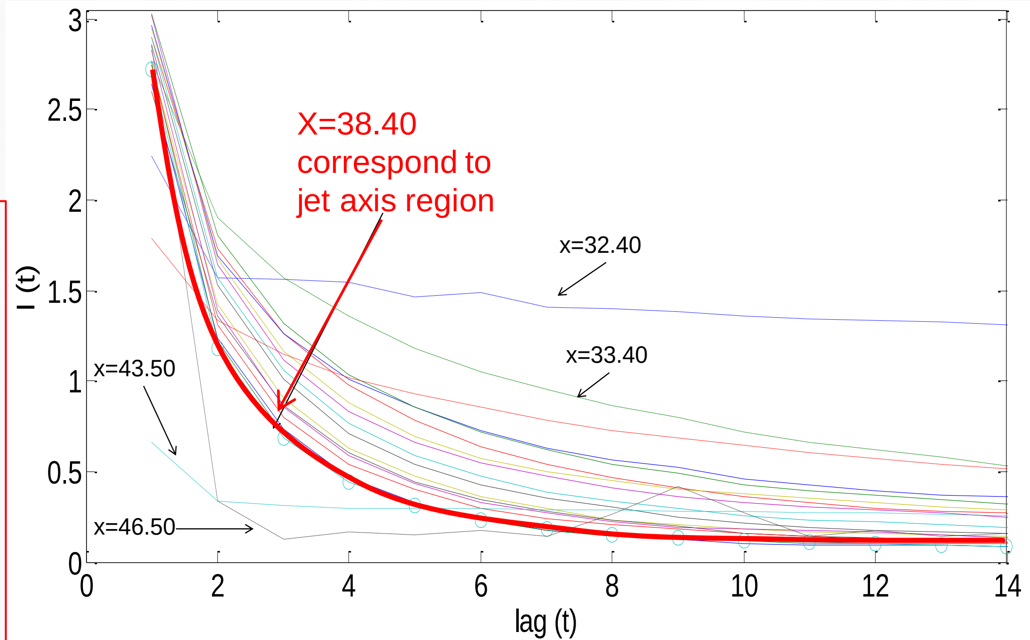


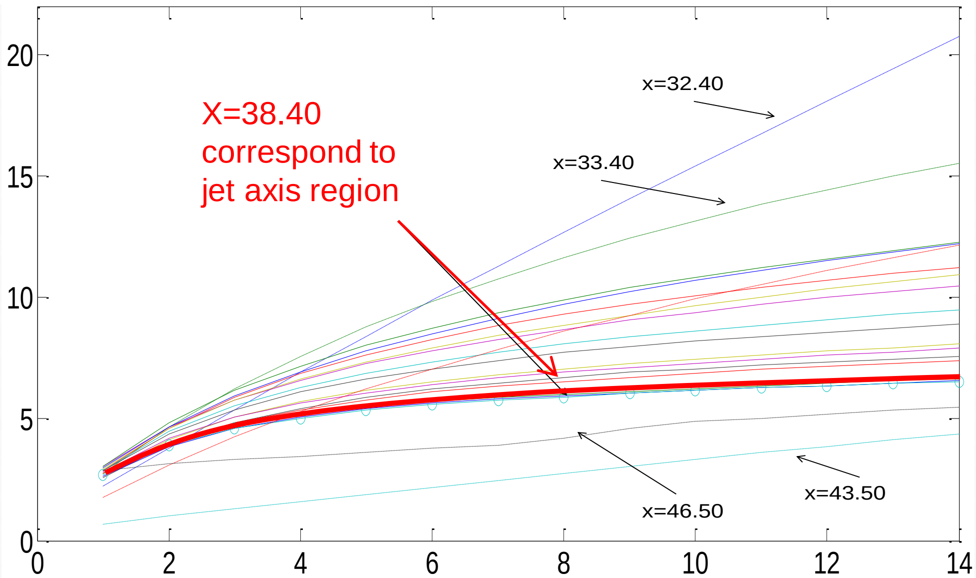


Case 1

✓ *Mutual Information*

- Lowest values for the center line region
- We know that near the jet centerline turbulence is fully developed and there appear many short-lived small scale turbulent structures, while near the jet boundary the large scale flow structures live longer
- At the jet axis region the memory of the flow structures is lost fast while at locations close to the boundaries memory lasts longer.



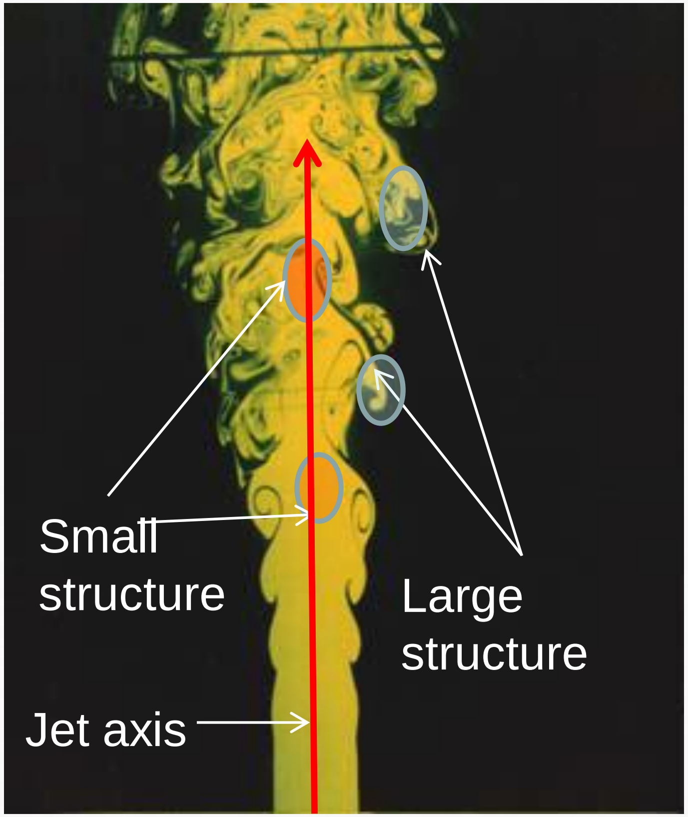


✓ *Cumulative Mutual Information*

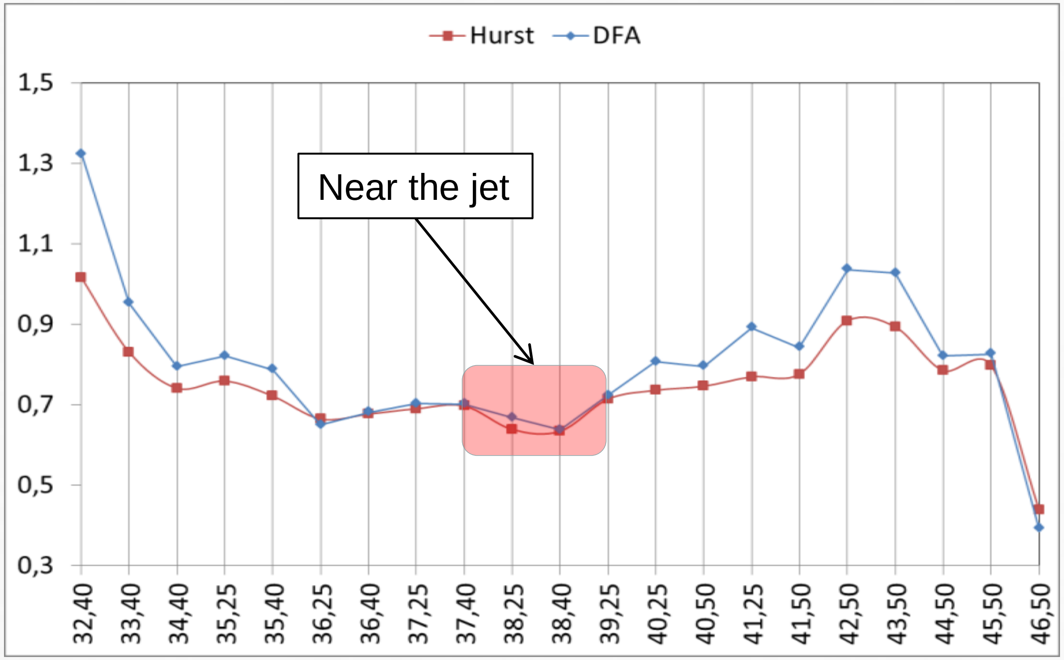
Turbulent jet

- ✓ *Near to the jet axis the turbulence is fully developed and in this region there appear many short-lived small scales turbulent structures appear, while we move towards from the inner to boundary the large scale flow structures live longer*

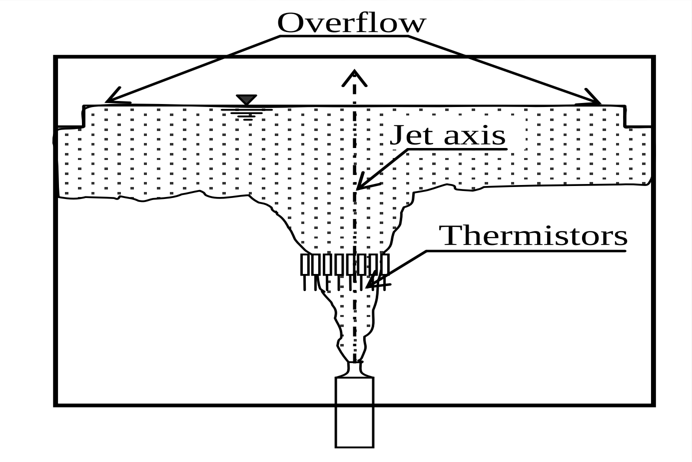
- ✓ *The lowest values of the measures are expected close to the jet region and especially in the jet axis since in this region there are many short-lived and small vortexes while presence of long-life large vortexes exist close to the boundaries*



Case 1 - *Hurst exponent*

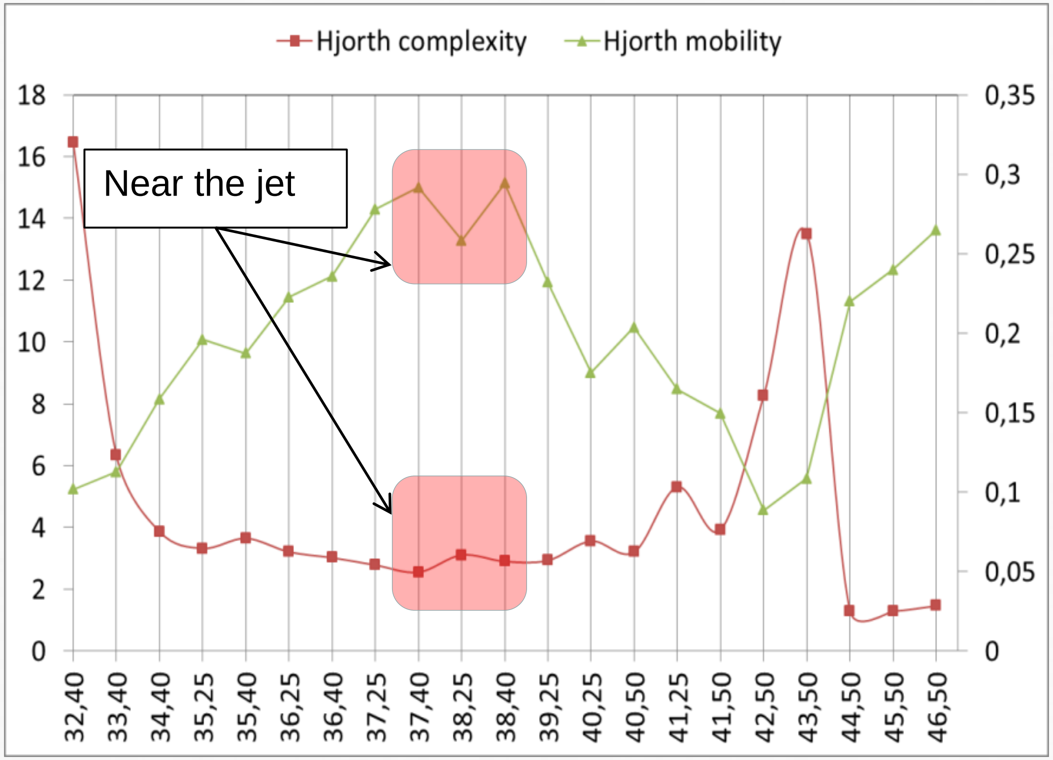


Time series along the horizontal axis

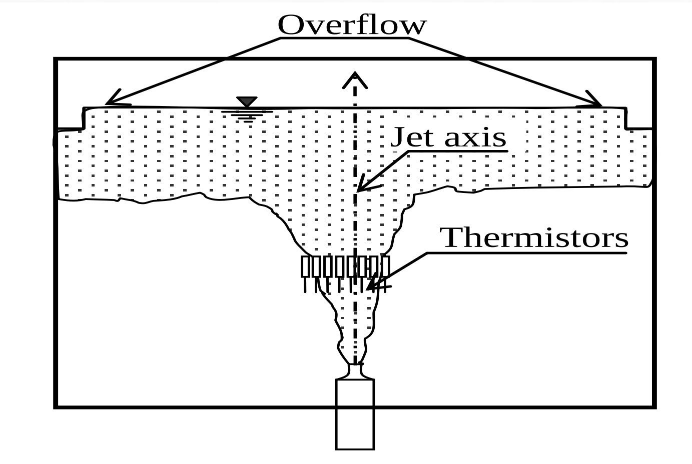


- In general persistent behaviour (H>0.5)
- Lower persistence region corresponding to the jet axis region since the short living structures destroy long memory
- At more distant positions larger and longer living structures result in high values

Case 1 - *Hjorth Mobility & Complexity*



Time series along the horizontal axis



- Mobility larger in the central region. Mobility is showing how fast the variance varies in a time series, i.e. faster in the centre slower at larger distances- this is related to the short living structures present in the axis region.
- At more distant positions larger and longer living structures result in high values
- Complexity captures changes in the stages of a time series

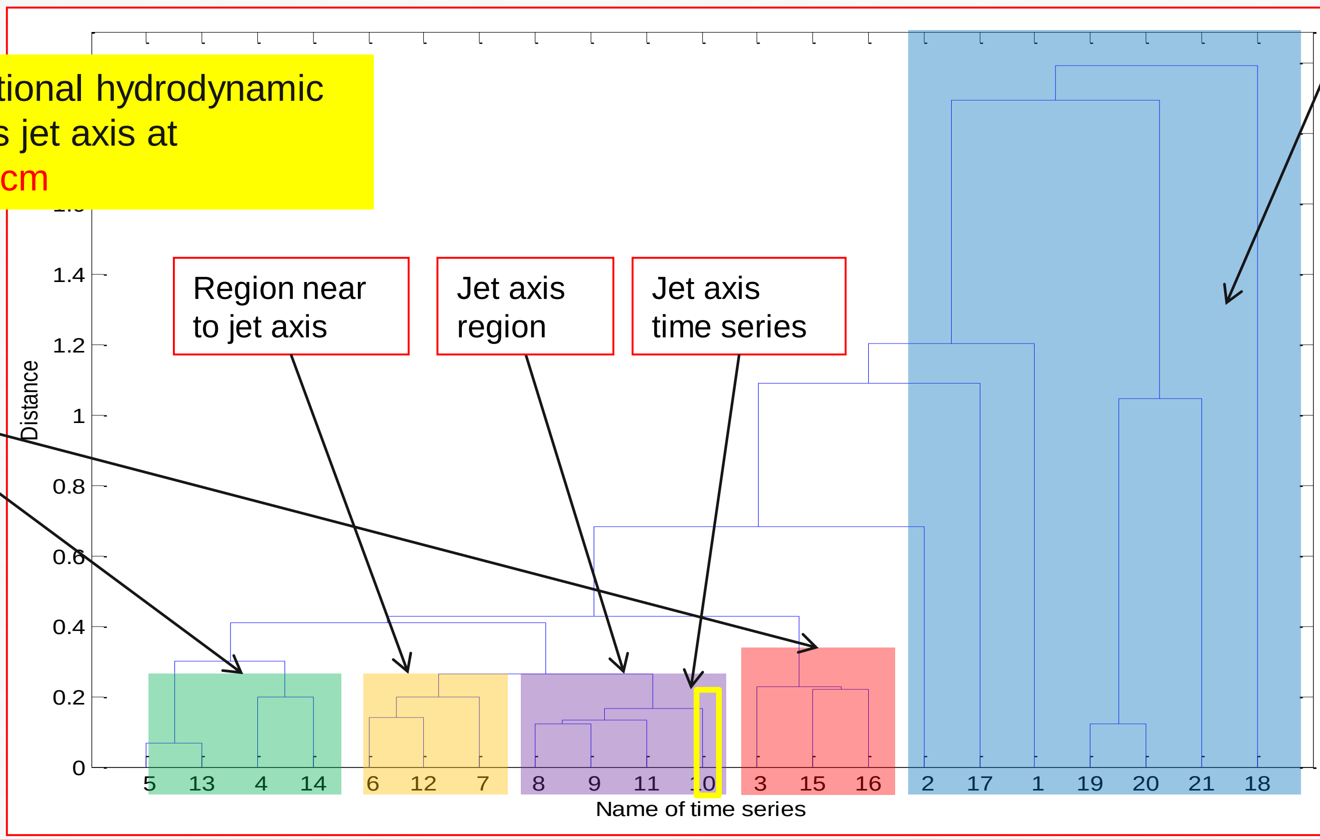
Case1 - Clustering

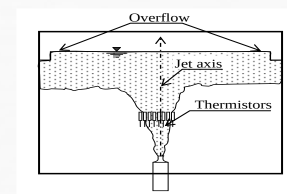
Identifying different regions of the jet

Shape of nozzle	T_o(°C) jet	T_a(°C) ambient
Round 1,5cm	58.60	24.60

Name of time series	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Location at the horizontal axis	32,4	33,4	34,4	35,25	35,4	36,25	36,4	37,25	37,4	38,25	38,4	39,25	40,25	40,5	41,25	41,5	42,5	43,5	44,5	45,5	46,5

Conventional hydrodynamic methods jet axis at x=37.75cm

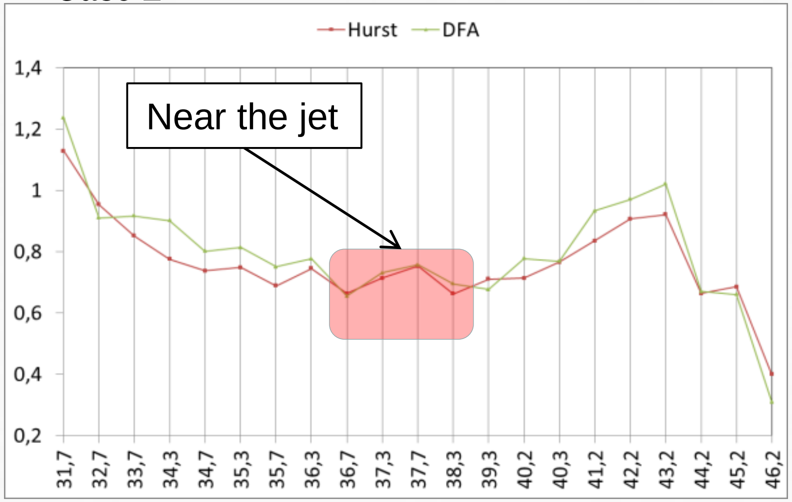




Other Cases

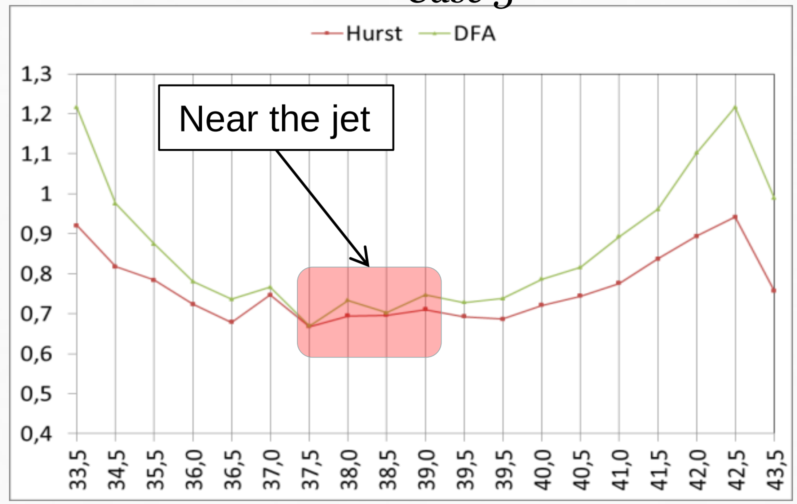
Hurst exponent

Case 2



Time series along the horizontal axis

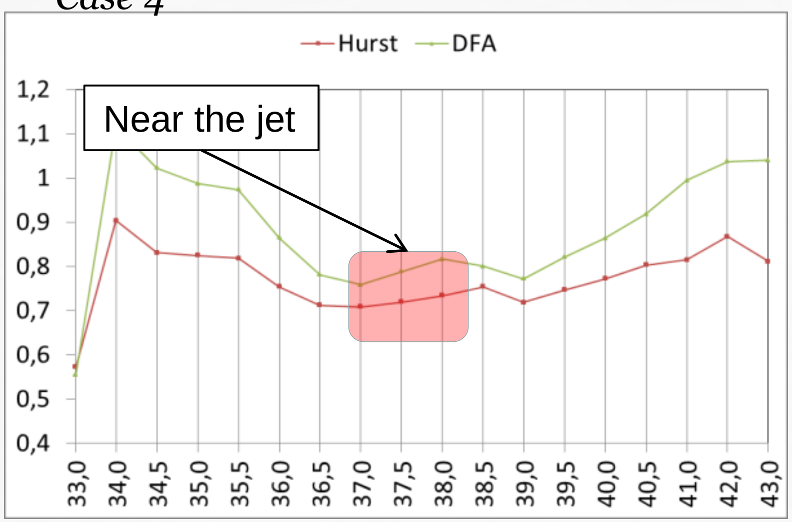
Case 3



Time series along the horizontal axis

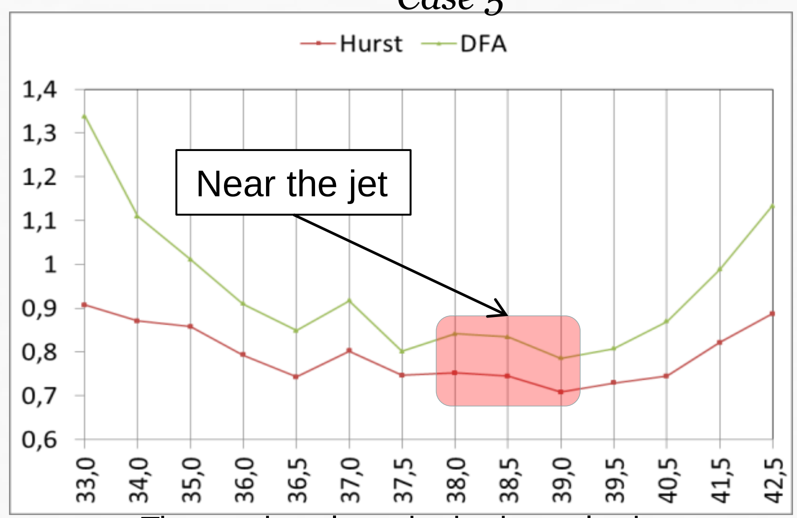
Lowest values

Case 4



Time series along the horizontal axis

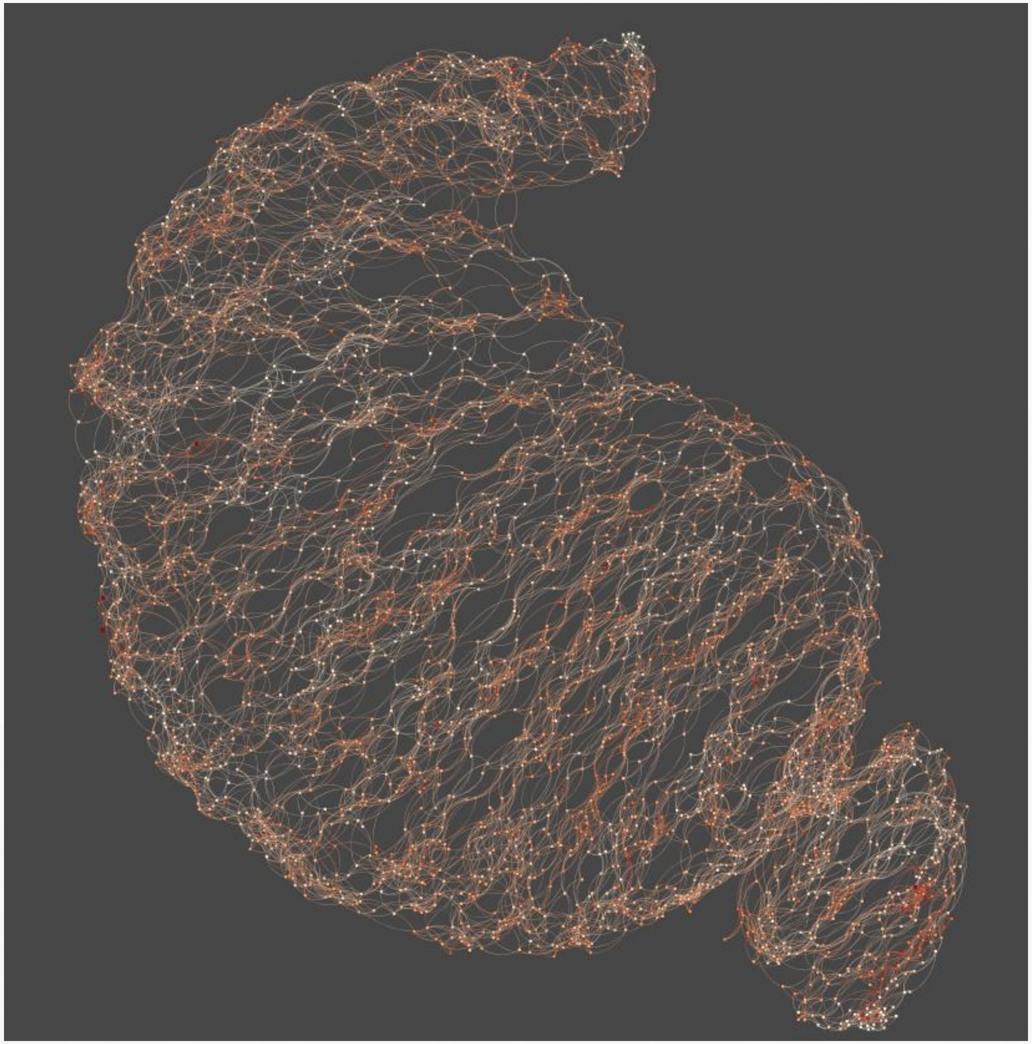
Case 5



Time series along the horizontal axis

Complex Networks

Transforming time series into Complex Networks



Seems a promising technique

Can be further employed in the future for spatiotemporal analysis

Complex Networks

Some notions

- ✓ *A Network (graph) G=(V,E) consists of a set of nodes (V) that are interconnected with links or edges (E)*

- ✓ *A Network of N nodes can be described by the NxN adjacency matrix* $A=[a_{ij}]$

$a_{ij}=1$ *if the link* $i-j$ *exists,*

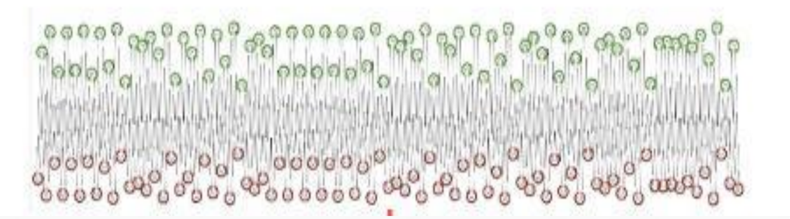
$a_{ij}=0$ *otherwise*

$$A=\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Complex Networks and time series

Construction of the Complex Networks (Xiaoke Xu et. al (2008)

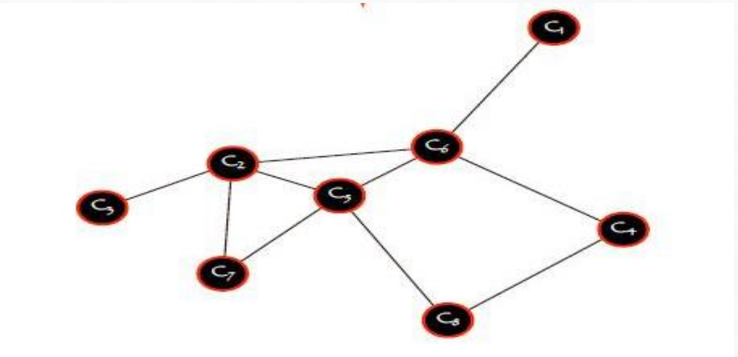
- ✓ *Time series*



- ✓ *Embedding the time series in an appropriate phase space and take each phase space point as a node in the network*
- ✓ *Select a fixed number of nearest neighbors for each point (node) and connect each point with its neighbors to form a complex network*

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	...
C_1	1	0	0	0	0	1	0	0	
C_2	0	1	1	0	1	1	1	0	
C_3	0	1	1	0	0	0	0	0	
C_4	0	0	0	1	0	1	0	1	
C_5	0	1	0	0	1	1	1	1	
C_6	1	1	0	1	1	1	0	0	
C_7	0	1	0	0	1	0	1	0	
C_8	0	0	0	1	1	0	0	1	
...									

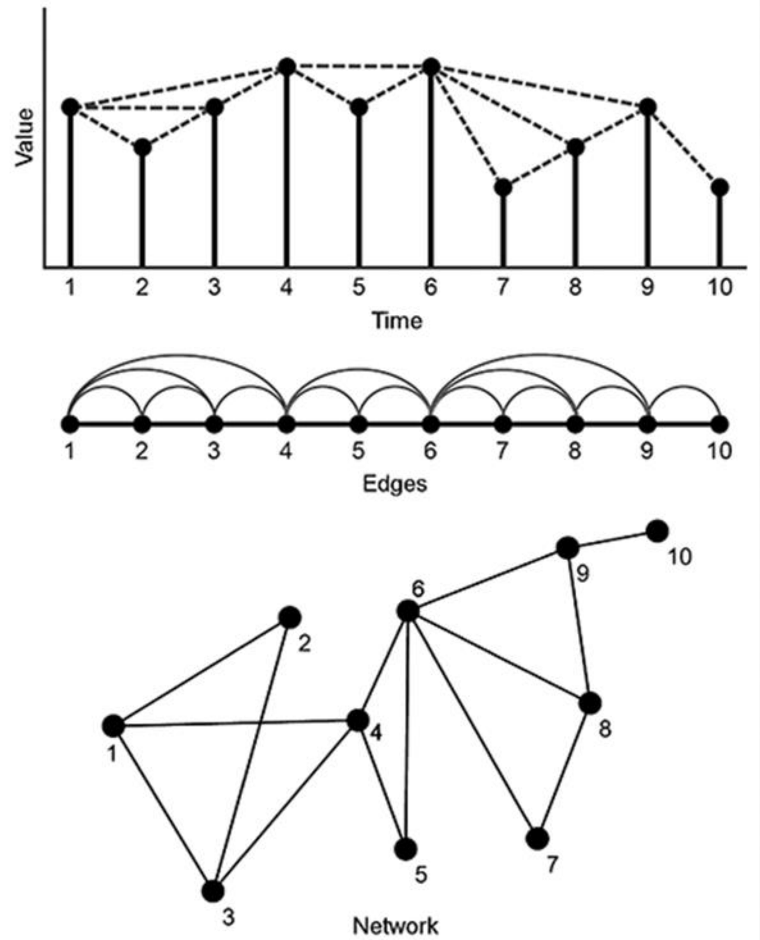
- ✓ *Construct the adjacency matrix*
- ✓ *Construct the complex network*



Complex Networks Time Series

Construction of the Complex Networks (L. Lacasa et. al (2008))

Visibility graph method:



Let x(ti)i=1,…N is the time series

.

Two nodes x(ti) and x(tj) in the time series have visibility and become two connected nodes in the associated graph, if any other data (tk, x(tk)) placed between them (ti<tk<tj) fulfills

$$x(t_k) < x(t_i) + (x(t_j) - x(t_i)) \frac{t_k - t_i}{t_j - t_i}$$

i and j are connected if one can draw a straight line in the time series joining the two points i and j, such that, at all intermediate points (ti<tk<tj), x(tk) falls below this line

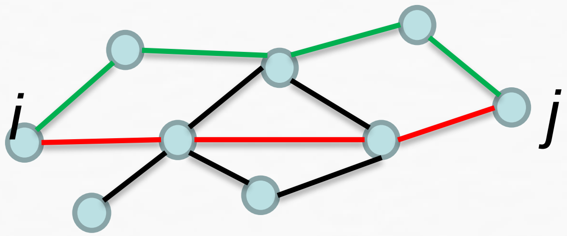
Complex Networks

Properties of Complex Networks

- ✓ *Shortest path (d_{ij}): Corresponds to the minimal distance between all paths that connect nodes i and j*

  d_{ij} is the red line (3) and

  not the green one (4)



- ✓ *Average path length (l): Is the average number of steps along the shortest paths for all possible pairs of network nodes. It measures the efficiency of information or mass transport on a network.*

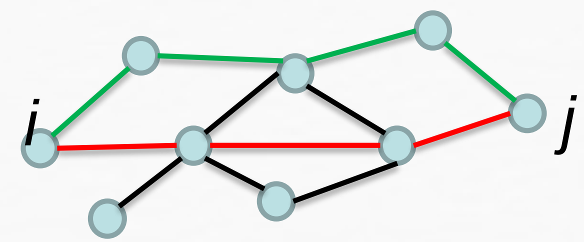
$$\langle d_{i,j} \rangle = \frac{1}{N(N-1)} \sum_{i,j} d_{i,j}$$

Complex Networks

Properties of Complex Networks

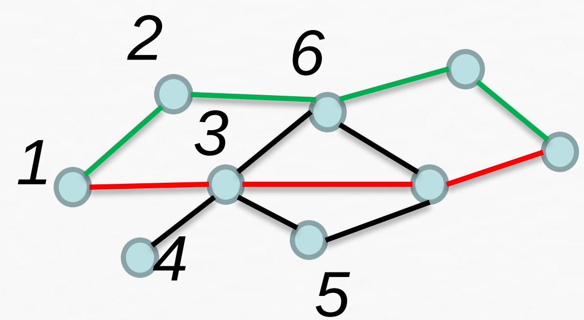
- *Diameter (D): The maximum length between all shortest paths*

  $D = max(d_{ij}) = 3$



- *Degree (Ki): The degree Ki of a node i is the number of connections of the node to other nodes*

  $K_1=2, K_2=2, K_3=5, K_4=1, K_5=2, K_6=4$

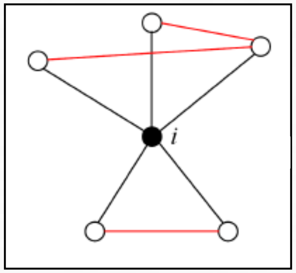


Complex Networks

Important properties of Complex Networks

- *Degree distribution:* *P(k) of a network specifies the fraction of nodes having exactly degree k*

- *Clustering coefficient:* *Ci is the ratio between the number of links E connecting the nearest neighbors of i and the total number of possible links between these neighbors.*



$$c_i = \frac{2e_i}{k_i\left(k_i - 1\right)} \qquad K_i = 5,\ e_i = 3$$

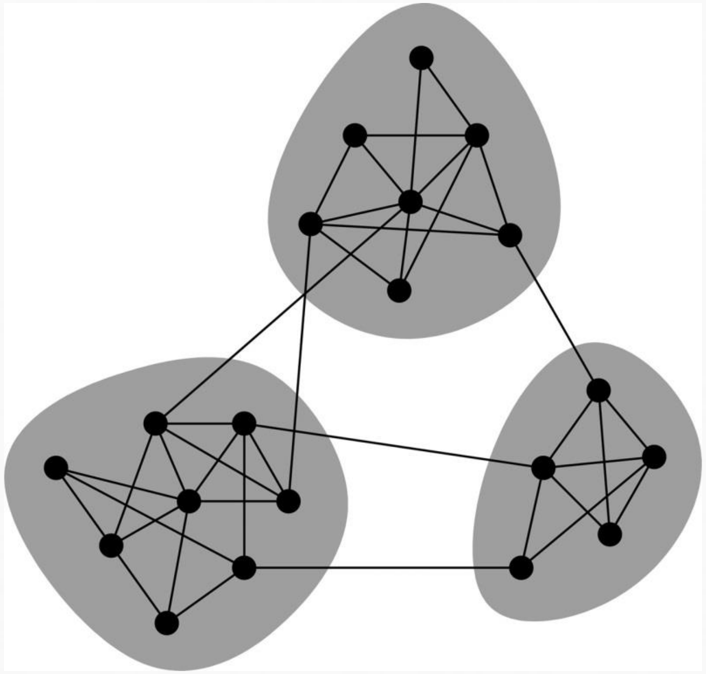
K_i is the degree of i, e_i is the number of links directly connecting neighbors of i

The clustering coefficient of a network C is the average of Ci over all nodes $C = \langle c_i \rangle = \frac{1}{N}\sum_i c_i$

Complex Networks properties

- ✓ *Modularity (M): a measure of the structure of a network. It measures the strength of division of a network into modules (also called groups, clusters or communities).*

- ✓ *Networks with high modularity have dense connections between the nodes within modules but sparse connections between nodes in different modules.*



Newman M E J PNAS 2006;103:8577-8582

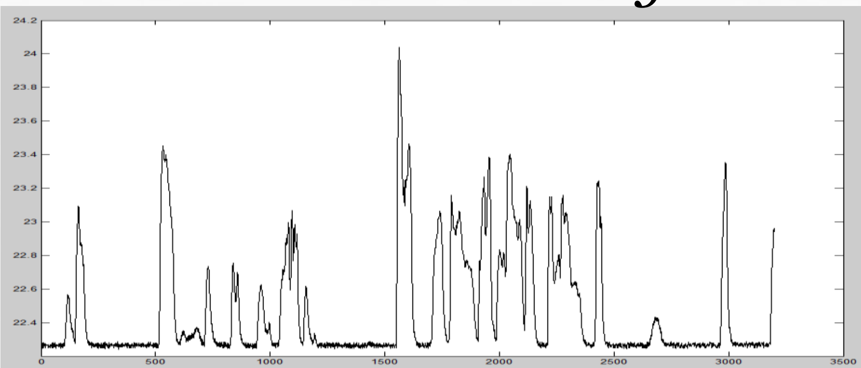
Complex Networks

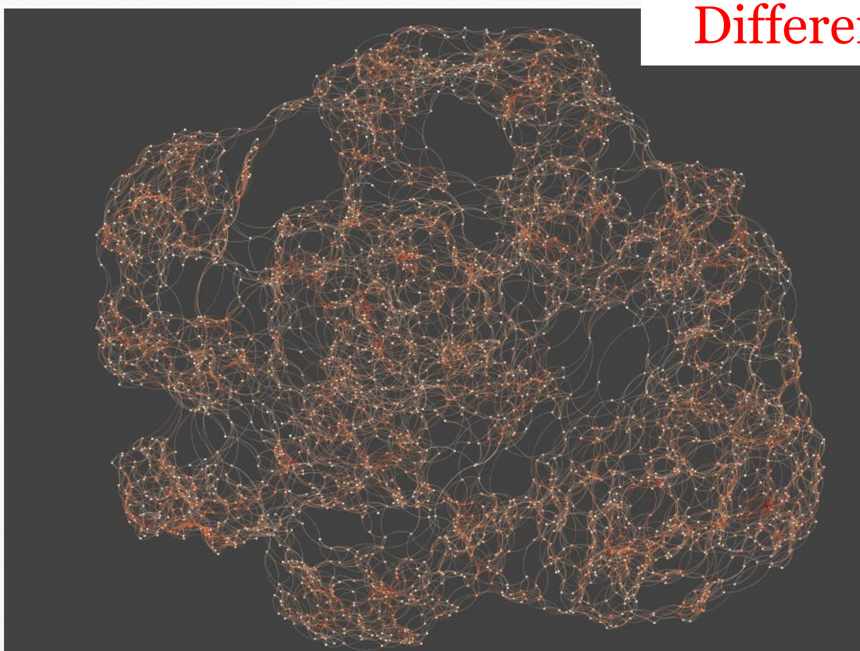
- ✓ *Main Idea: Different dynamical systems demonstrate distinct local structures in phase space.*

- ✓ *Time series with different dynamic (recorded from different region of the jet) exhibit distinct topological network structures*

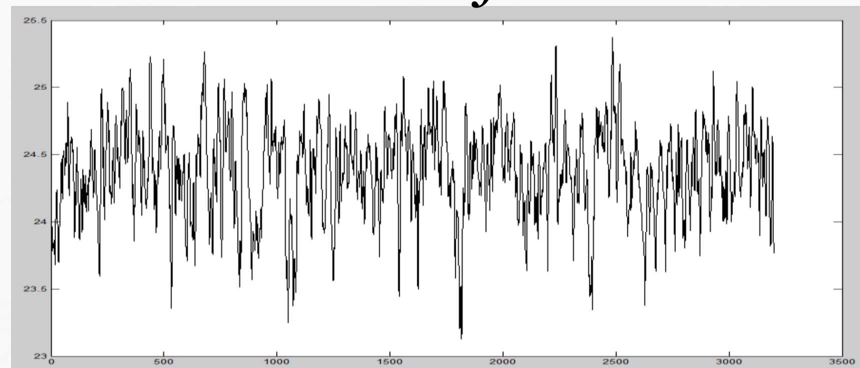
Complex Networks: Results

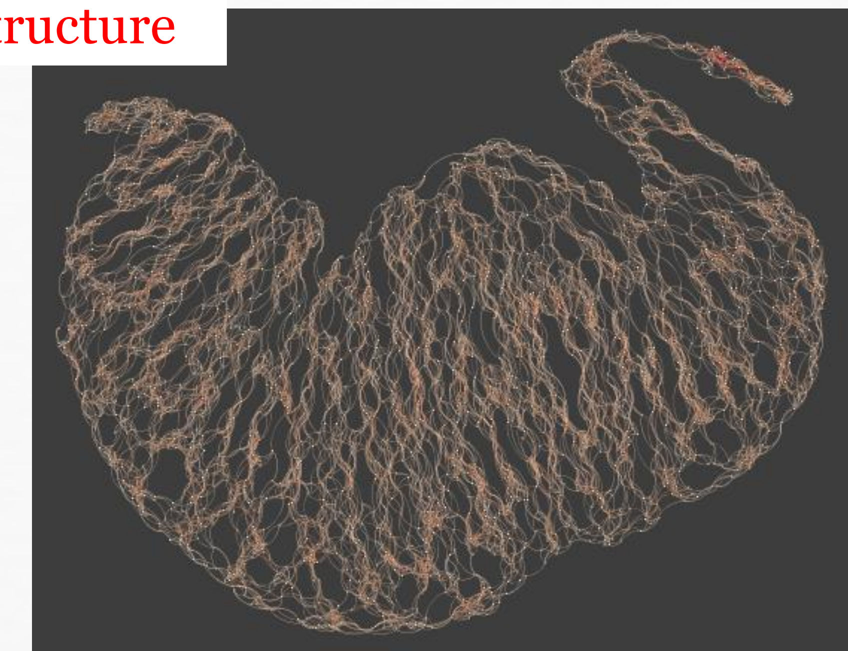
Near the boundary





Near the jet line

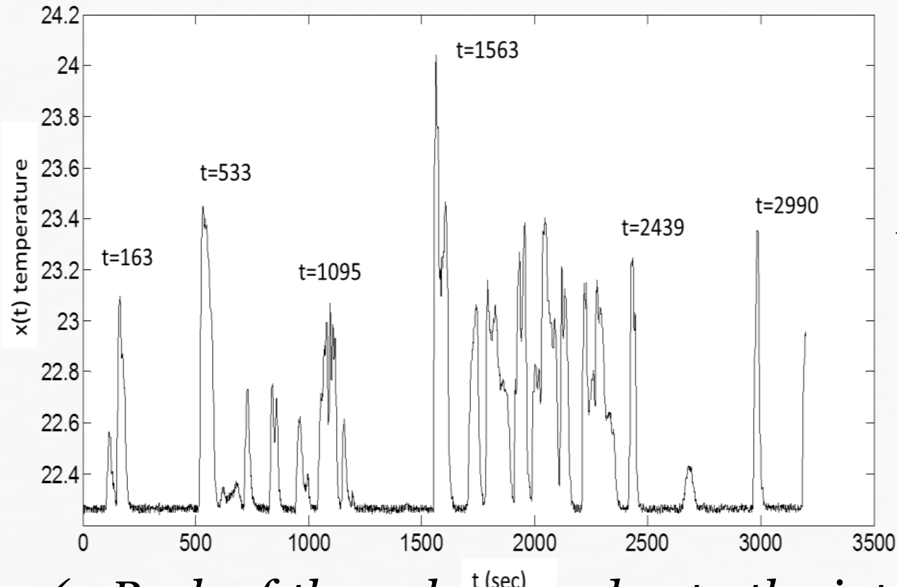


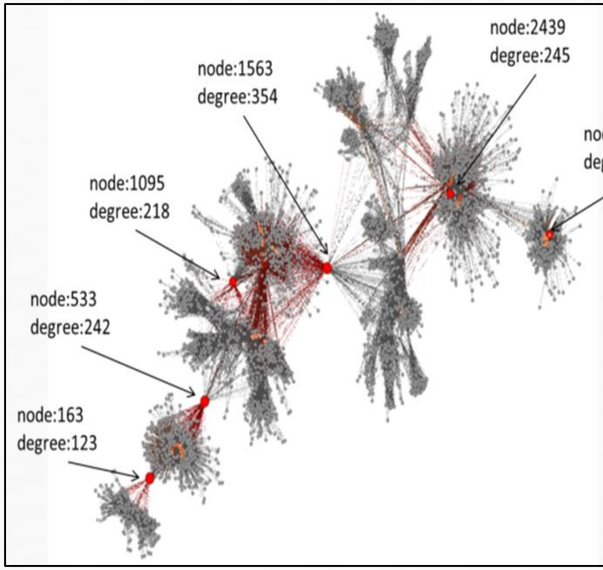


Different structure

Complex Networks: Results

- *Visibility method*

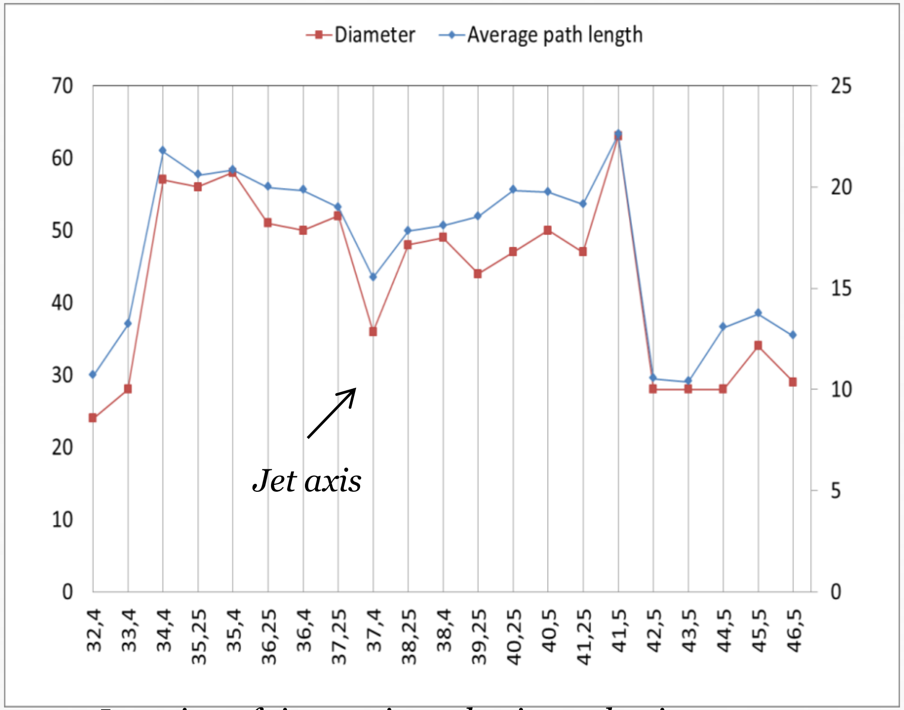




Node	Degree
163	123
533	242
1095	218
1563	354
2439	245
2990	165

- ✓ *Peak of the value are due to the interaction of small heated eddies to the bigger vortex*
- ✓ *Throughout the development of the time series if there exists a peak and the previous and next value would not be located very close this data tend to have higher degree than the other data.*
- ✓ *At the network each point represented as a hub with different degree*
- ✓ *Physically this means that at these points we have a strong influence of a heated vortex to less heated fluid*

Complex Networks: Results



Location of time series at horizontal axis

Case 1

Lowest values of

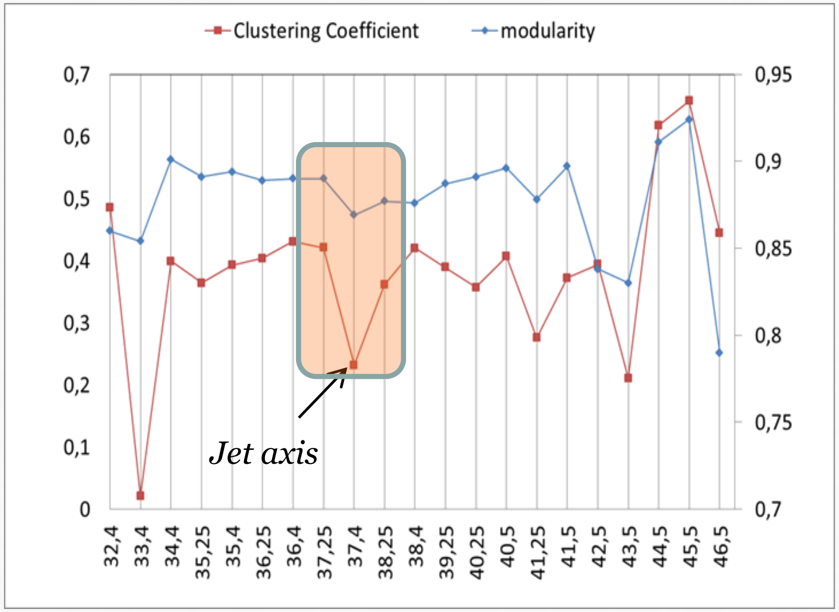
Diameter : The maximum length between all shortest paths
Average path length (l): average number of steps along shortest paths for all possible pairs of network nodes

In the jet axis region : fully developed turbulence → increased presence of short-lived small scales → change of states occur faster → successive states are less linked

As we move towards to boundary large scale longer living structures persist → change of states take more time to occur

Complex Networks: Results

Case1



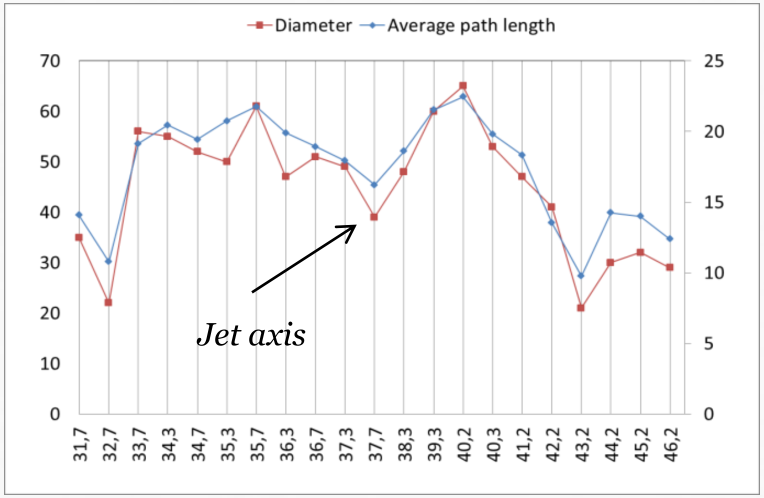
Location of time series at horizontal axis

Clustering coefficient: Ci is the ratio between the number of links E connecting the nearest neighbors of I and the total number of possible links between these neighbors.

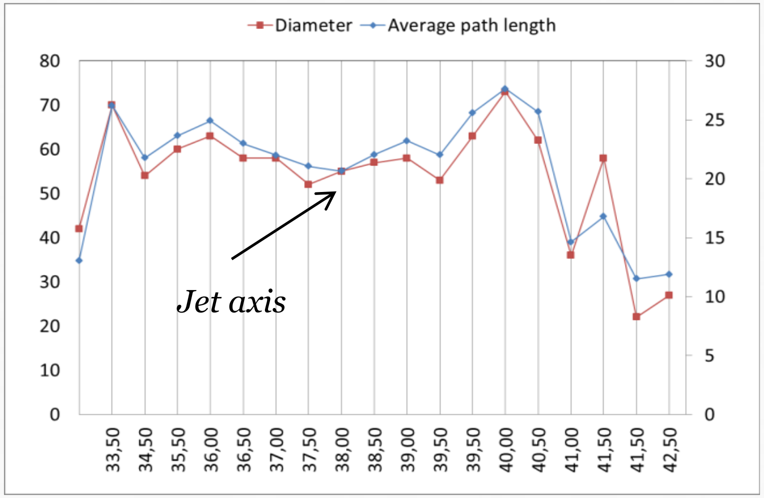
Due to presence of short lived small structures connections between successive sates are reduced compared to regions where long-lived structure persist.

Modularity (M): a measure of the structure of a network. It measures the strength of division of a network into modules (also called groups, clusters or communities).
Short live structures result in fewer well defined separations. More interconnections are present.

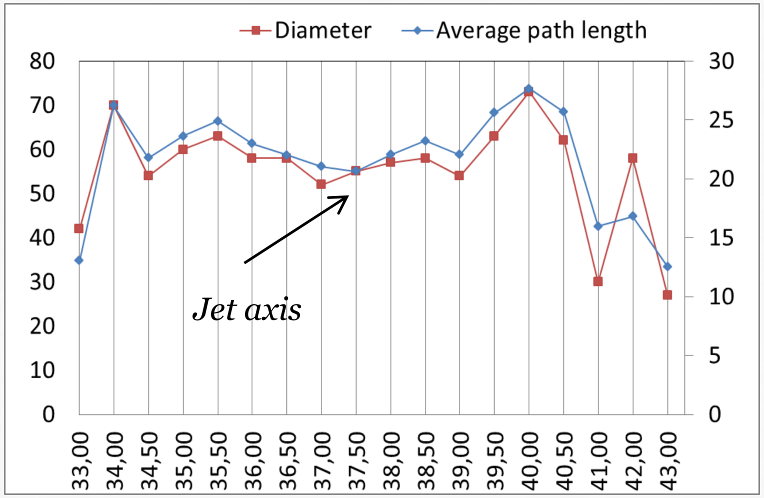
Other Cases- *Diameter and Average path length*



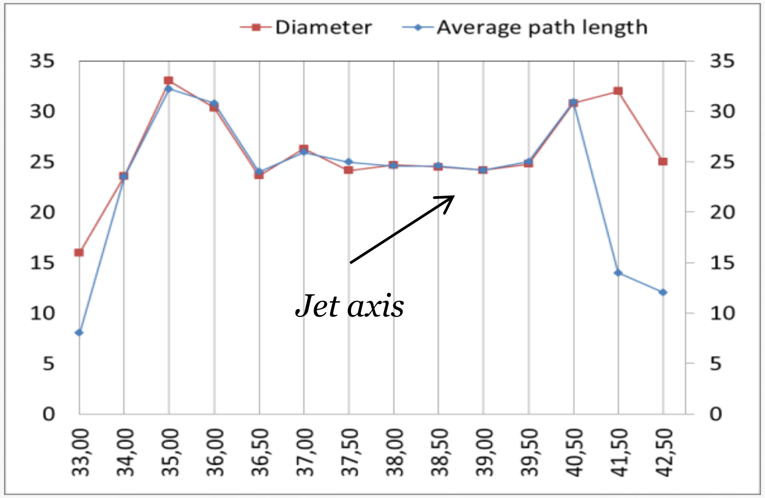
2nd Case Study



3rd Case Study



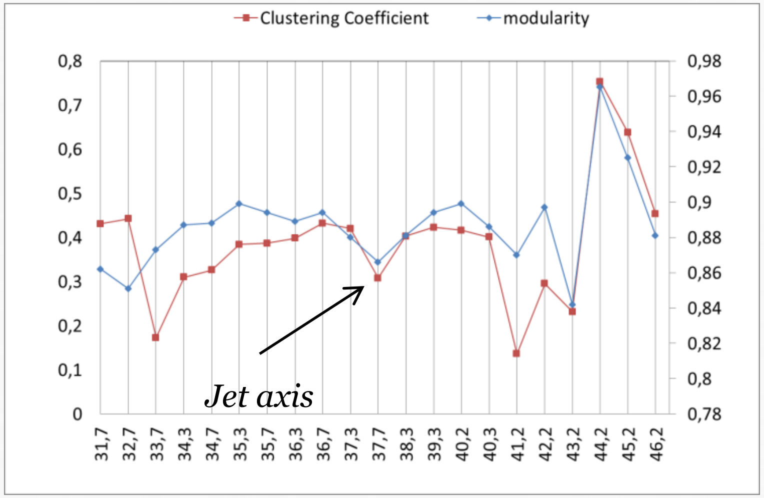
4th Case Study



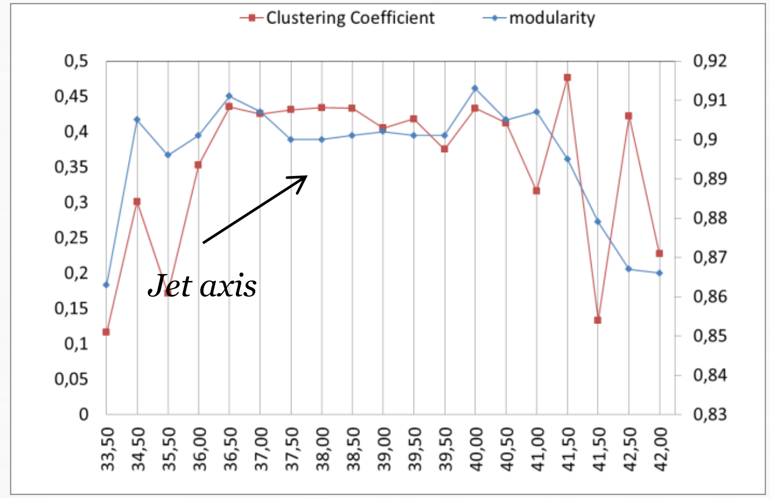
5th Case Study

Case Studies

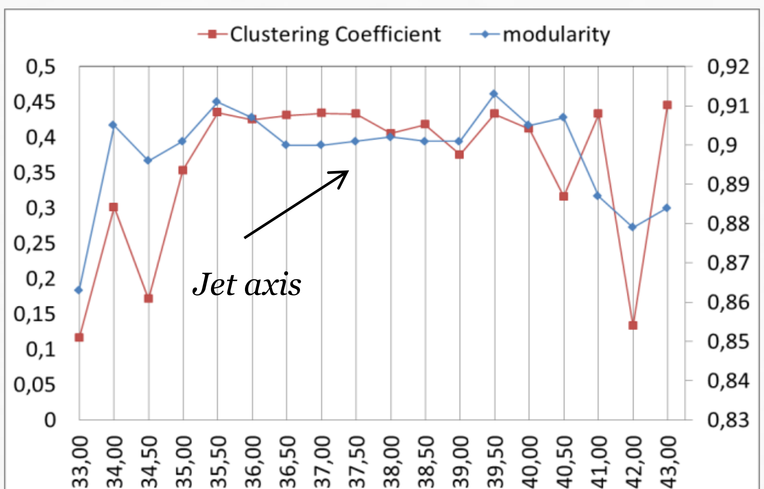
Clustering Coefficient and Modularity

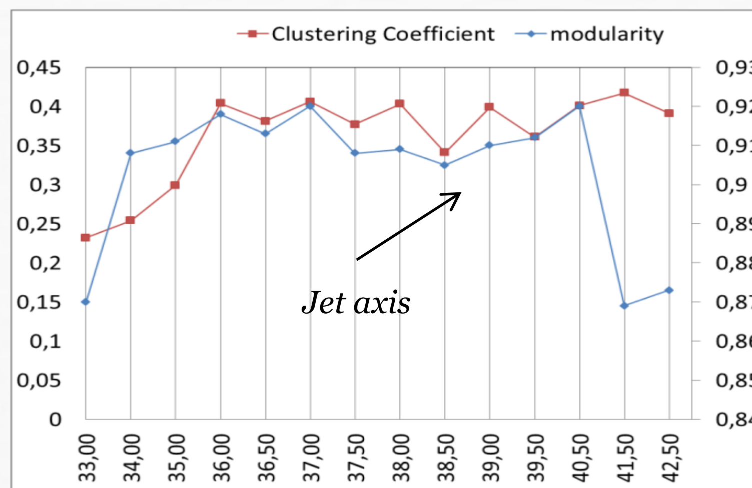


2nd Case Study



3rd Case Study





4th Case Study

5th Case Study

Conclusions

Results of jet axis location using our methodology and comparison with estimations from hydromechanics methods

Case Study	Shape of nozzle	Measurement station attributed to the jet axis using the clustering procedure (present study)	Estimated location of jet axis using a Gaussian fit
1st	Round	38.25	37.75
2nd	Round	37.70	38.00
3rd	Elliptical	38.00	38.00
4th	Elliptical	37.50	38.00
5th	Elliptical	39.00	38.20

Conclusions

Aim: Distinguish the jet axis region from the regions near the boundary (ambient water) and the intermediate regions

- ✓ Various measures can provide information about various regions of the jet as well as about the location of the jet axis
- ✓ Using a combination of all the measures along with a clustering procedure can discriminate far better various regions of the flow based on a different behavior and lead to a methodology for obtaining the location of jet axis
- ✓ Analysis is capable of extracting information and can be useful for a more clear discrimination of the time series near jet axis from others that correspond to the region near the boundaries
- ✓ This methodology seems quite promising for application in complex flows, as well as in applications where several different state zones exist in a physical system where one can have access only to spatiotemporal data
- ✓ The time series derived from different regions along the horizontal line, exhibit different topological features of the network

THANK YOU

FOR YOUR ATTENTION

Questions?