Synchronization Phenomena and Chimera States in Networks of Coupled Oscillators

A. Provata

National Center for Scientific Research "Demokritos", Athens

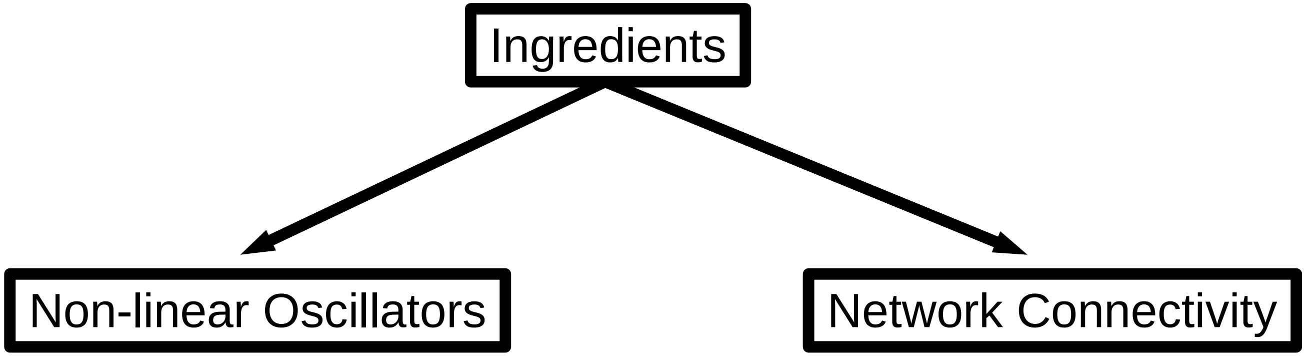
24th SummerSchool-Conference on "Dynamical Systems and Complexity"

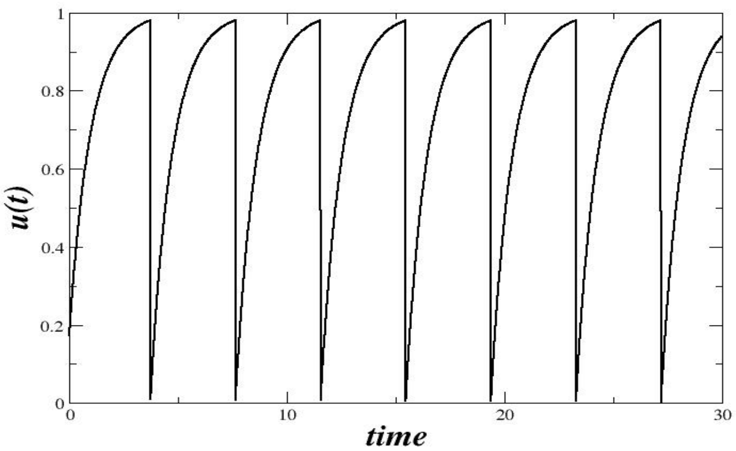
Volos, July 12-21, 2017

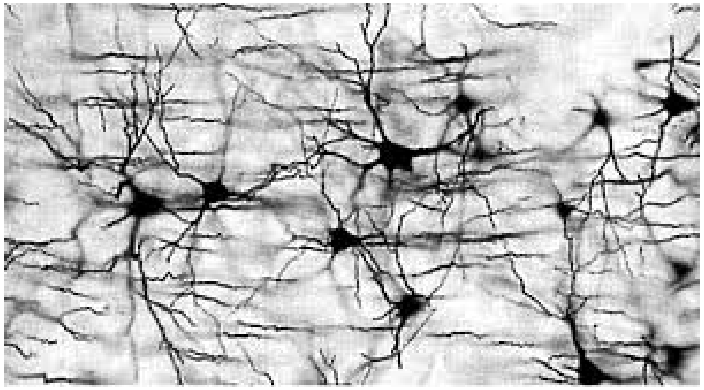
Overview:

1. Introduction & Motivation
   - The System: Network and Dynamics
   - Network: [by Brain MRI Imaging]
   - Dynamics and Synchronization phenomena:
   - What is a chimera state?
   - Applications in Brain Science et al.
3. The Leaky Integrate-and-Fire (LIF) Model
   - Nonlocal Connectivity
   - Other connectivities (Reflecting, Diagonal)
   - Hierarchical Connectivity
   - Non-local connectivity 2D & 3D
3. The FitzHugh Nagumo (FHN) Model
   - Non-local Connectivity 1D
   - Hierarchical Connectivity
4. Conclusions & Open Problems

1.1 Neuron Network and Dynamics





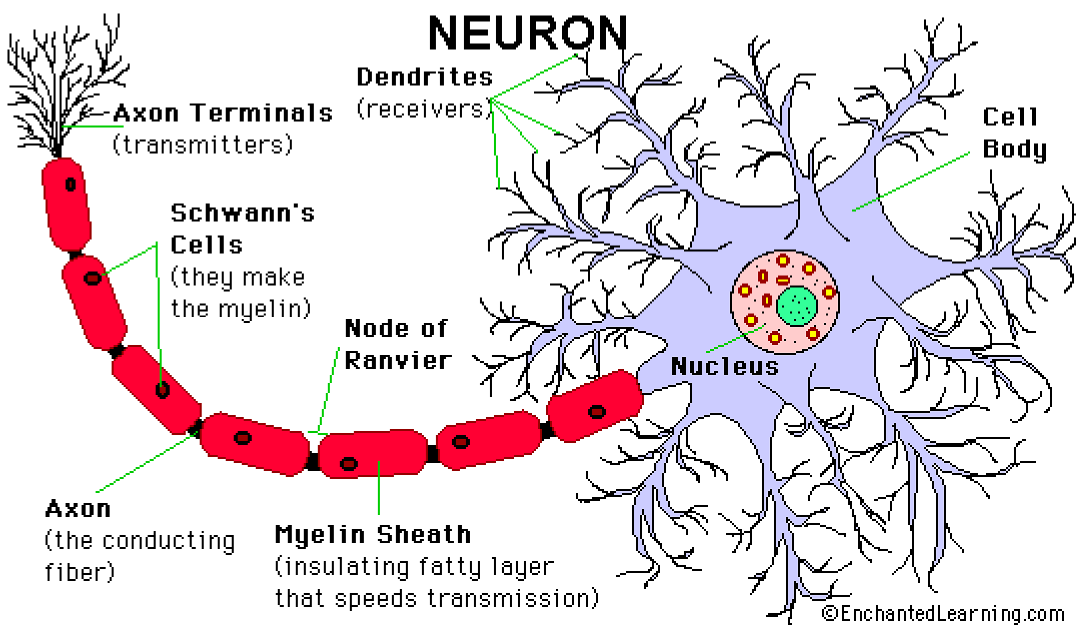


$$\frac{du_i(t)}{dt} = f[u_i(t)] \quad + \quad \sum_{j=1}^{N} \sigma_{ij}[u_j(t) - u_i(t)]$$

1.2 Brain Connectivity Structure

-The brain contains ***neurons*** which are electrically excitable cells which process and transmit information through electrical signals:

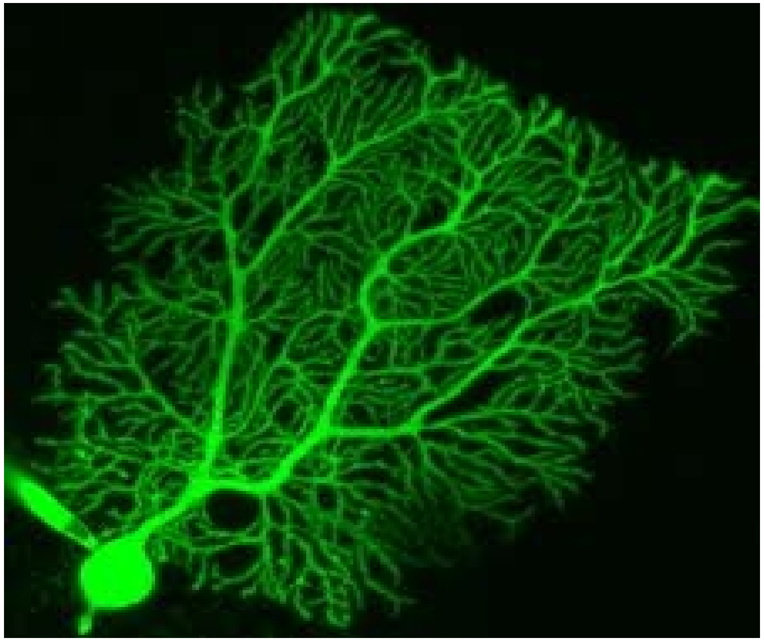
- **~2 10^{10} neurons** in the human brain (4 10^{6} in the rat brain)
- **7 000 synapses** (connections) of each neuron with others
- ***soma:*** 4-100 μm, contains the nucleus
- ***dendrites:*** extensions with many branches, receive signals
- ***axons:*** (10-...-1000) X (soma size), connect neurons and transmit signals (Usually neurons have 1 axon, but this axon usually splits and branches to undergo communication with many other target receiving neurons, kinetic neurons up to 1m!)
- ***axon terminals:*** contain synapses,specialised structures where neurotransmitter chemicals are released to communicate the signal to the other cells)

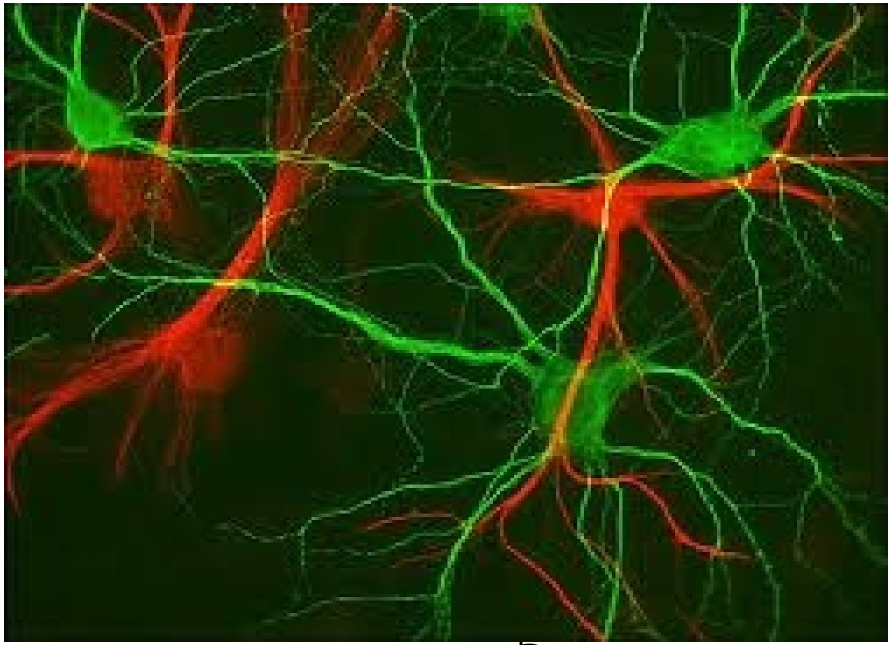


Basic Functions:

-The brain contains ***neurons*** which are electrically excitable cells which process and transmit information through electrical signals:

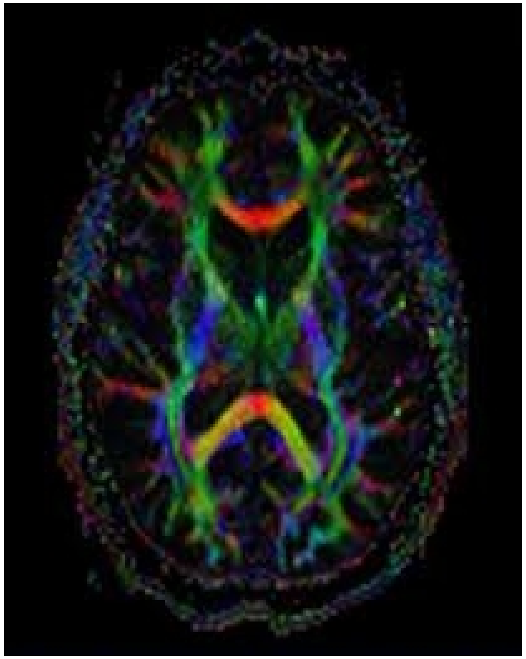
- ***soma*** (contains the nucleus, typical 25μm)
- ***dendrites*** (receive signals)
- ***axons*** (connect neurons and transmit signals, size 1μm, max 1m!)
- ***axon terminals*** (contain synapses to communicate the signal)





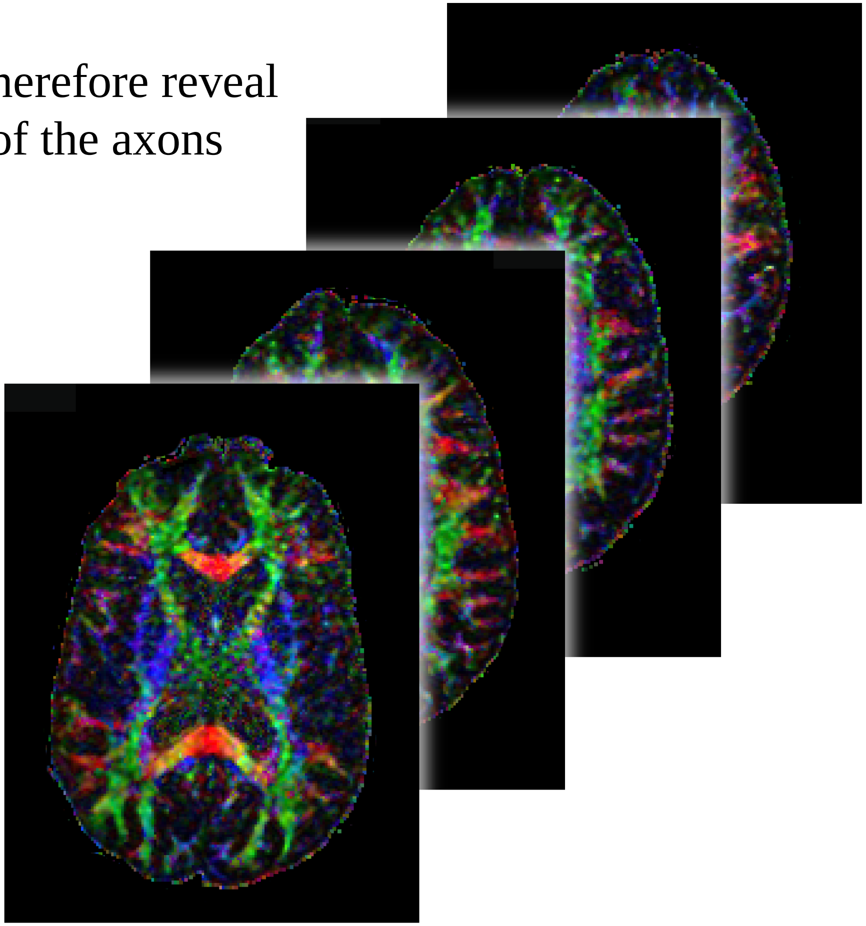
1.3 Diffusion Tensor Images of the Human Brain (DTI-MRI)

- DTI-MRI is a technique which allows the visualisation of the ***neuron axons network*** of the brain, based on the diffusion of the water molecules around the axons.
- non-invasive
- allows for detection of abnormalities and diseases.
-no need for radioactive tracer injection usually.



Basser PJ et al., Biophys. Journal (1994);
-idem J. Magn. Resonance Imaging (1994);
Mori S. & van Zijl PCM, Fiber Tracking (2002).

Molecular diffusion in tissues is not free in brain tissue, but it reflects interactions with many obstacles. One of these obstacles is the axons.
Water molecules move easier in the direction parallel to the axons, than perpendicular to them.
Water molecule diffusion patterns can therefore reveal microscopic details about the structure of the axons and indicate normal or diseased states.



Water motion => Axons visualisation

- DTI-MRI is a technique which allows the visualisation of the diffusion of the water molecules in living tissue.

-Red, Green, Blue: Indicate the water diffusion in three directions: x,y,z

-Diffusion weighted imaging DWI: the colour intensity (weight) indicates the rate of water diffusion at that location

-These images enable us to reconstruct the ***neuron axons network*** of the brain.

DTI – MRI: Neuron axons in 3D representation

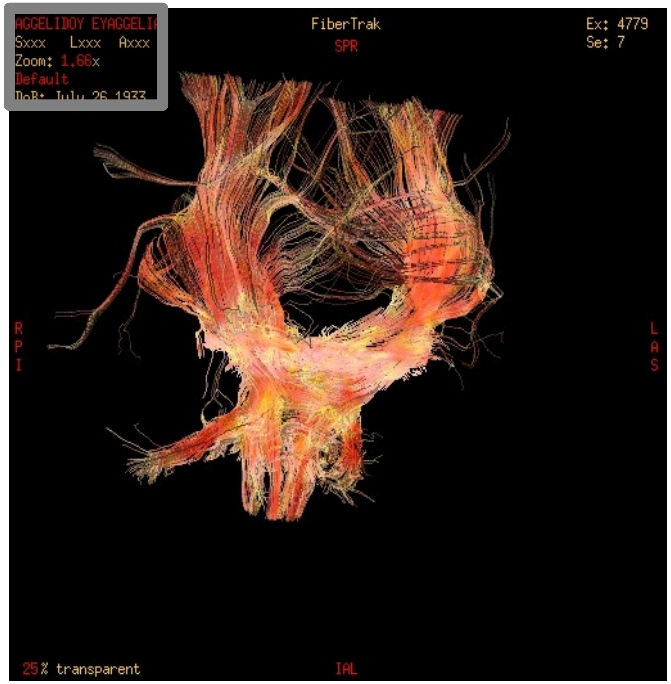
- With ***tractography*** the direction which corresponds to the maximum water diffusion designates the direction and connectivity of the neuron axons.

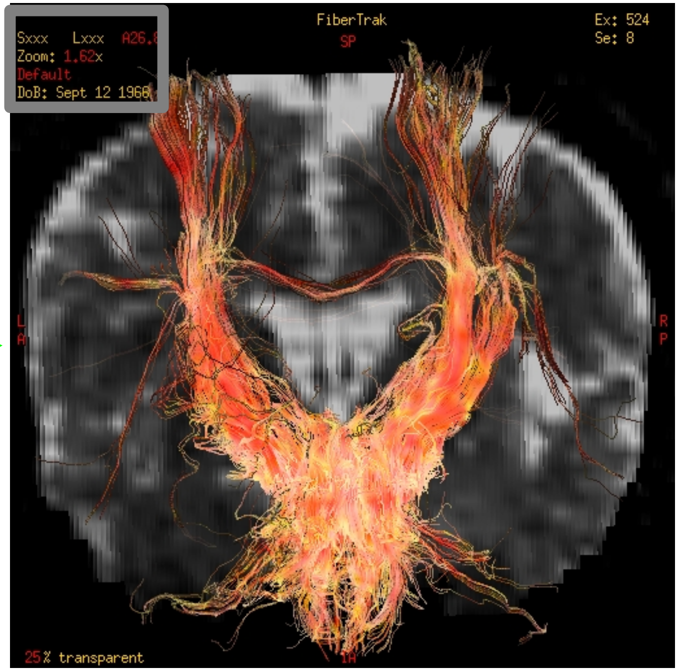
- Thus tractography helps in constructing the structure and connectivity of the network of the neuron axons in the brain.

-Used in the non-invasive diagnosis of brain diseases and traumas.

-Used to understand the brain functioning.

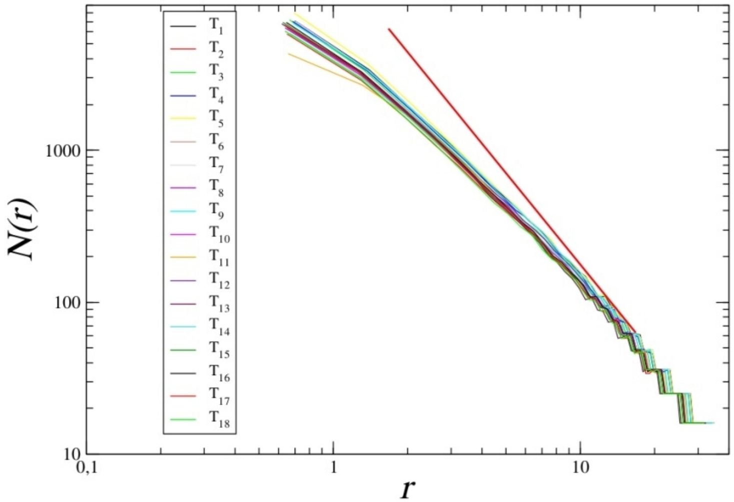
Region Of Interest (ROI): All neuron axons crossing a circular disk

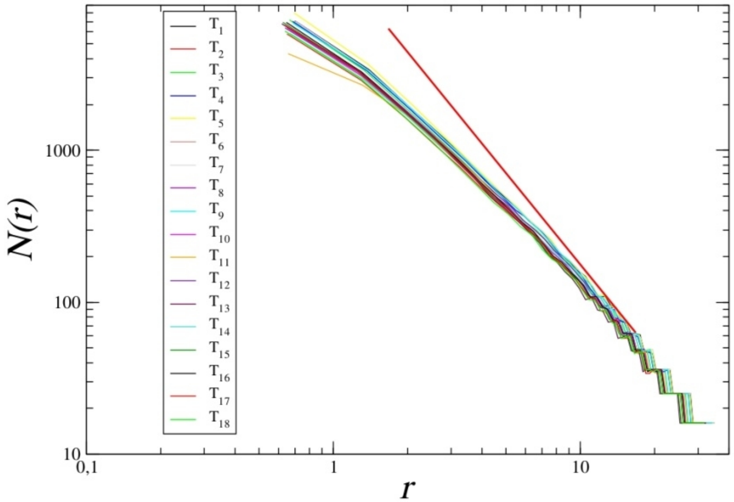




$$N(r) \sim r^{-d_f}$$

d_f=fractal dimension



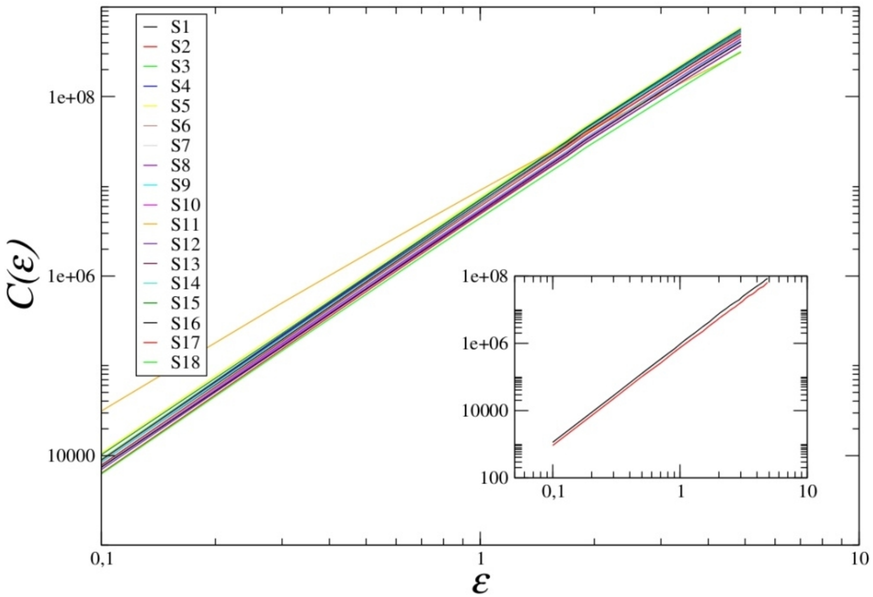


Fractal Analysis
Box-counting 2d
Red line: 2-d structure

Fractal Analysis
Box-counting 3d
Red line: 3d structure

Expert P. et al, J. Roy. Soc. (2011)
Katsaloulis P. et al, Fractals (2011)

df ~ 2.48



$$C(\epsilon) = \lim_{\epsilon \to 0} \frac{g(\epsilon)}{N^2}$$

$$g(\epsilon) = \sum_{i,j=1}^{N} \Theta(\epsilon - |\vec{p}_i - \vec{p}_j|)$$

$$C(\epsilon) \sim \epsilon^{d_{cor}}$$

Correlation Dimension: $\boldsymbol{d_{cor}=2.8}$

Variability in the fractal dimensions points out to ***multifractality***

Feder J., *Fractals* (1988);
Takayasu H., *Fractals in the Physical Sciences* (1990).

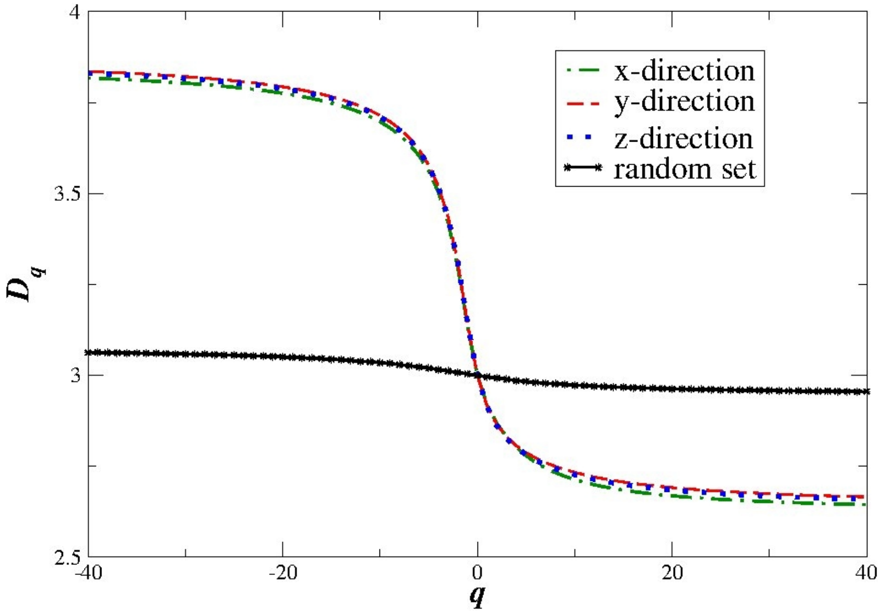
Multifractal representation on the NAN structure (local densities involved)

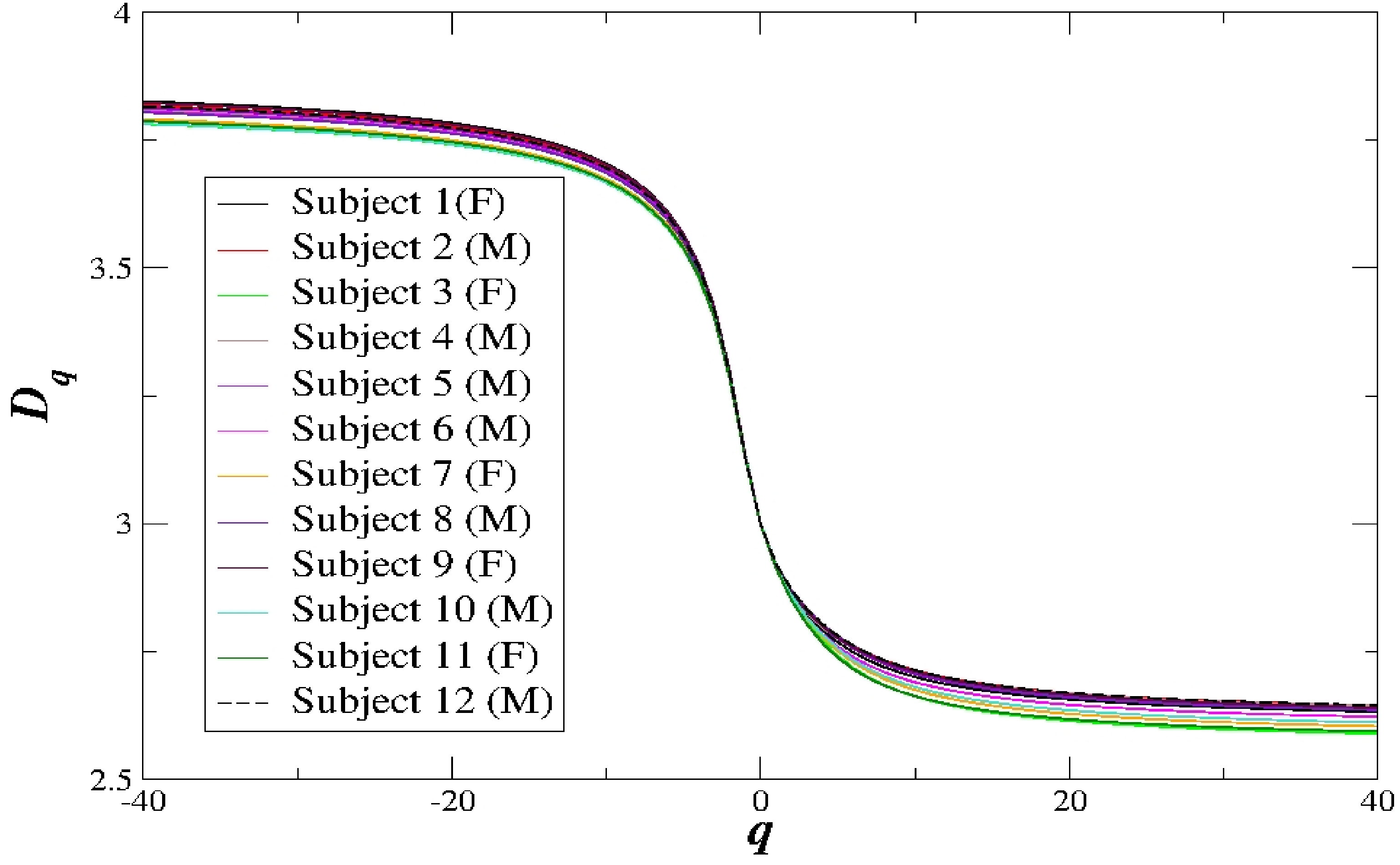
$$D_q = \lim_{r \to 0} \frac{1}{\ln r} \frac{\ln \sum_{i=1}^{N} P_i^q}{q-1}, \quad q \neq 1$$

$$D_1 = \lim_{r \to 0} \frac{1}{\ln r} \sum_{i=1}^{N} P_i \ln P_i$$

Divide the space in cells of size $\boldsymbol{r}$ with $\boldsymbol{r} \rightarrow \boldsymbol{0}$.

$\boldsymbol{P_i}$ is the fraction of the structure included in the cell $\boldsymbol{i}$.





Similarities in healthy male and female NANs

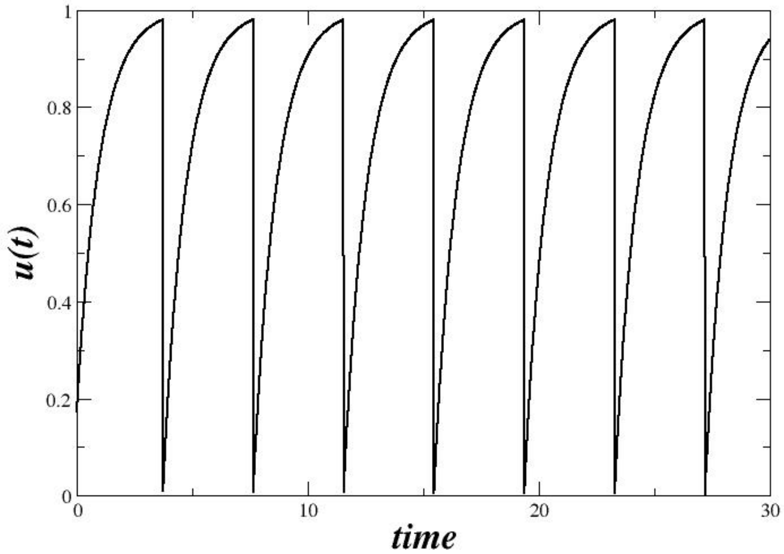
We know that Neuron Degenerative Disorders (Alzheimer, Parkinson, Schizophrenia et al) affect the connectivity of the brain:

Can the analyses as Fractal, Multifractal, Correlations, Connectivity patterns etc help in:

a) understanding the cause of these disorders?
b) predict their evolution?
c) design “biomarkers” for their monitoring
d) early detection of the disorders

=>> Properties of neuron network is crucial for brain dynamics

1.4 Dynamics of single neurons & Synchronization phenomena



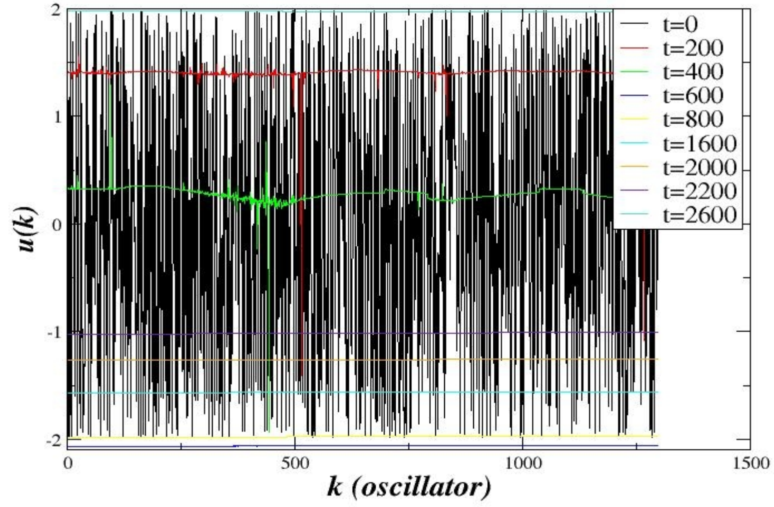
Single Neuron
!!!Spiking!!!

Coupled system
- Single frequency!!! or
- Distribution of frequencies and/or
- Distribution of parameters and/or
-Distribution of coupling constants

1.5 Synchronization Phenomena

1. Full synchronization:
   Starting from random initial states

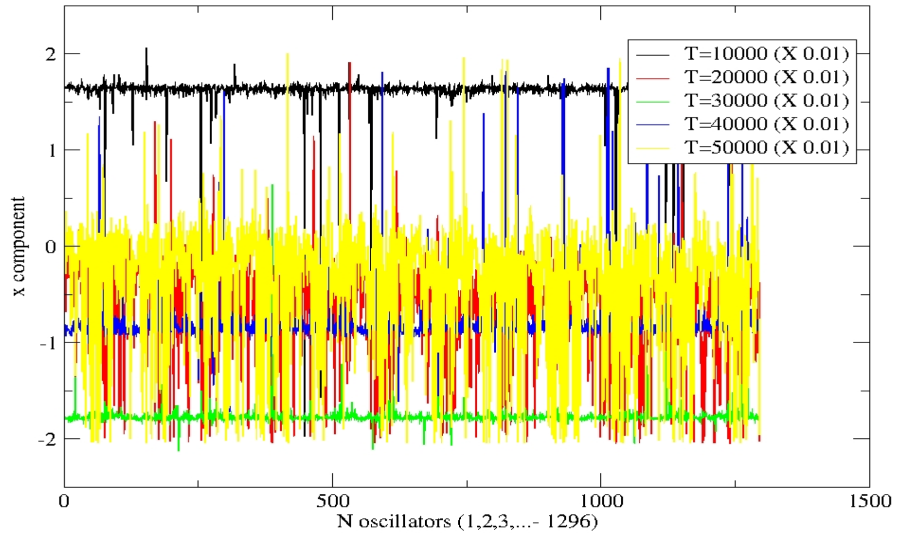
   $u_i(t=0) \neq u_j(t=0)$, $i,j=1,2,\ldots N$,
   $\exists\, t_0 : u_i(t) = u_j(t)\ \forall\, t\ \&\ \forall\,(i,j)$, for $t>t_0$



connectivity 614 random connections

2. No-synchronization:
   Starting from random initial states

   $u_i(t=0) \neq u_j(t=0)$, $i,j=1,2 \ldots N$,
   $\Rightarrow u_i(t) \neq u_j(t)\ \forall\, t$ and $\forall\,(i,j)$

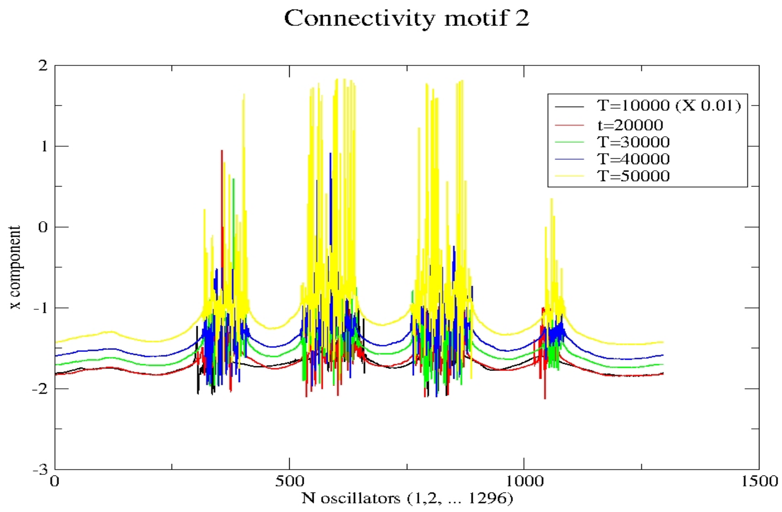


3. Partial synchronization:
Starting from random initial states
and identical oscillators & $\sigma_{ij} = \sigma$

$$u_i(t=0) \neq u_j(t=0),\ i, j = 1, 2 \ldots N$$

$$\exists\ t_0\ \&\ i1, i2, \ldots ik$$
$$: u_{ij}(t) = u_{il}(t)\ \forall\ t \text{ and } \forall\ (ij, il),\ \text{for } t > t_0$$

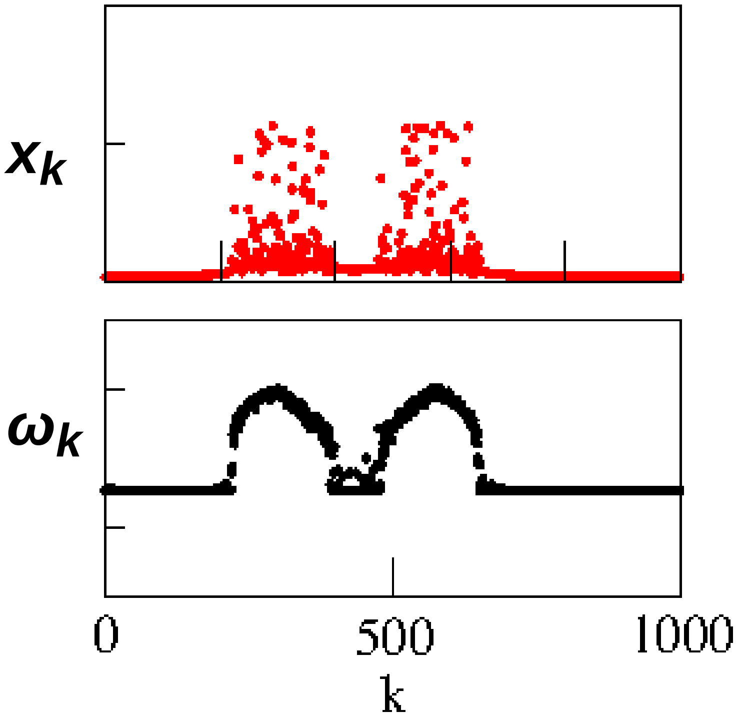
while
$$u_i(t) \neq u_j(t)\ \forall\ t \text{ and } \forall\ (i,j) \notin \{i1, i1, \ldots ik\}$$



1. 6 Elements of Chimera States

Elements:
- identical oscillators
- identically linked in networks
- random initial conditions

Outcomes:
*Complete Synchronization
++ Partial synchronization
(or partial disorder...)
"Chimera State"
*Complete disorder



-2002: Kuramoto and Battogtokh, Nonlin. Phen. in Complex Sys., 5:380.
-2004: Abrams and Strogatz, Phys. Rev. Lett., 93:174102.
-2015: Panaggio and Abrams, Nonlinearity, 28:R67 (review).
-2016: Schöll, EPJ-ST, 225:891 (review).

Named by: *Abrams and Strogatz in 2004*

Sphinx: chimeric creature with head of a human, body of a lion, and wings of a bird.

Greek: woman, malevolent
Egyptian: man, benevolent

Chimera: with head of a lion, body of a goat, and tail of a snake.

Red-figure Apulian plate, c. 350–340 BC

Centaur: chimeric creatures with upper body of a man and lower body of a horse, living in the region of Pelion mountain.

Centaur,
Athenian cup
6th B.C.,
Toledo Museum of Art

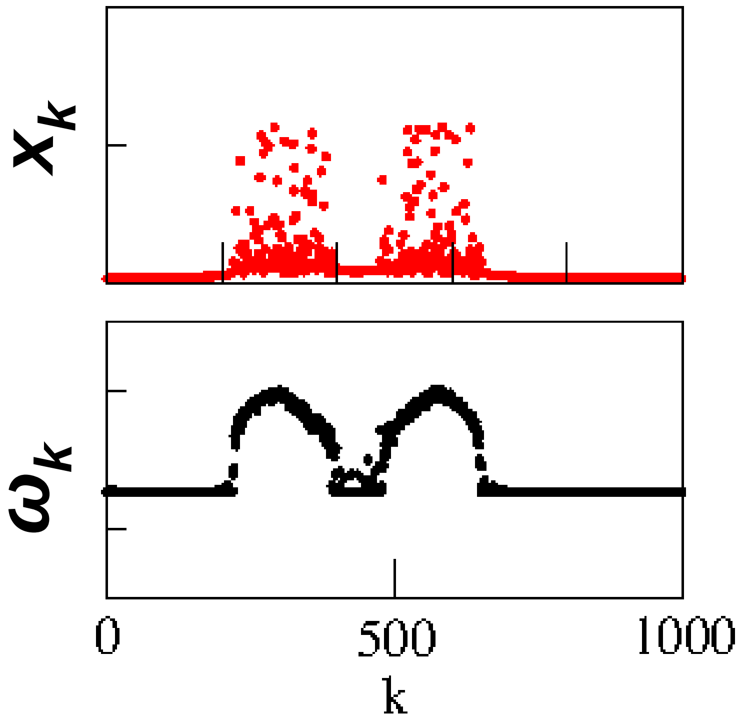
Quantitative Description

$$\omega_i = \frac{Number\ of\ cycles\ of\ element\ i\ \text{in}\ time\ \Delta T}{2\pi\ \ \Delta T}$$

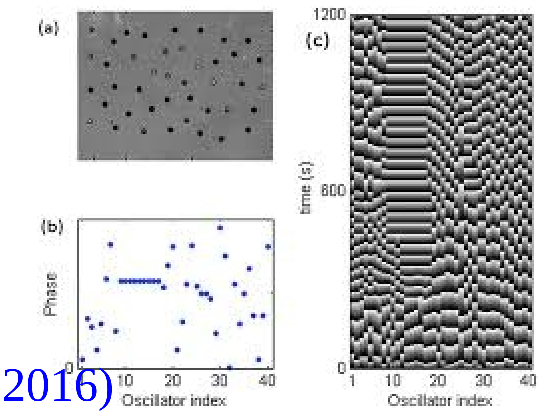
$$\Delta\omega = \omega_{incoh} - \omega_{coh}$$

$$N_{incoh} = \frac{1}{N}\sum_{i=1}^{N} \Theta\left(\omega_i - \omega_{coh} - c\right)$$

$$M_{incoh} = \sum_{i=1}^{N}\left(\omega_i - \omega_{coh}\right)$$



1.7 Experiments



Now it has experimental verifications in the domains:

*Mechanics: Coupled metronomes

(*Martens et al, Proc. Nat. Acad. Sciences, 2013*)

(Blaha, Burrus,… Sorrentino, Chaos, 2017)

*Electronics: Equivalent circuits

(Meena et al., Int. Jour. Bifurcations and Chaos, 2016)

(Klinshov … Nekorkin, Phys. Rev. E, 2016)

*Chemical Dynamics: BZ experiments

(*Tinsley …. Showalter, Nature Physics, 2012*),

(Taylor … Showalter, Phys.Chem.ChemPhys. 2016).

.

*Lasers: Optical coupled-map lattices via liquid-crystal spatial light modulators

(*Hagerstrom et al., Nature Physics, 2012*)

(Viktorov, Habruseva, ...Kelleher, CLEO-IQEC-2013).

*Uni-hemispheric sleep in birds and dolphins (*Panaggio and Abrams, 2015*)

* Partial and mal-functionality of the brain (*Tsigkri et al, 2016, Isele et al., 2016*)

* Synchronization phenomena in the firing of fireflies etc *(Ott, Antonsen, Chaos 2017)*

1.8 Applications in Neuron Dynamics

Partial Synchronisation in the form of Chimera States is first numerically observed in the domain of neuron dynamics:

* Phase Oscillator (*Kuramoto et al. 2002, Abrams et al. 2004*)
* FitzHugh Nagumo Oscillator (*Omelchenko et al, 2013, 2014, 2015*)
* Leaky Integrate-and-Fire (*Olmi et al., 2010, Luccioli et al. 2010, Tsigkri et al. 2015*)
* van der Pol oscillators (*Ulonska et al., 2016*)
* Hindmarsh-Rose Oscillator (*Hizanidis et al., 2014, 2016*) ……..

Population Dynamics & Reaction Diffusion:
* BZ Reaction: (T*insley …. Showalter, Nature Physics, 20120)*
*Population Dynamics *(Hizanidis … Provata, PRE 2015)*

Materials:
* Metamaterials: *(Lazarides et al., PRE 2015)*

Importance & Influence of :

a) Dynamics	b) Network Topology
Spiking	Nonlocal Connectivity
Cut-offs	Topology of connections
System parameters	Coupling strength

2.1 The Leaky Integrate-and-Fire Model (Louis Lapique, 1907)

[propagation of electrical signals in neurons, simple, popular in computational neuroscience]

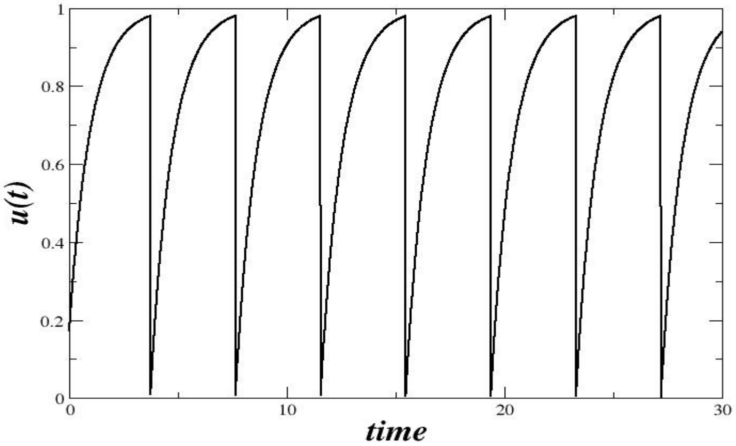
$$\frac{du(t)}{dt} = \mu - u(t)$$

$$u(t) \rightarrow 0, \quad when \quad u(t) > u_{th}$$

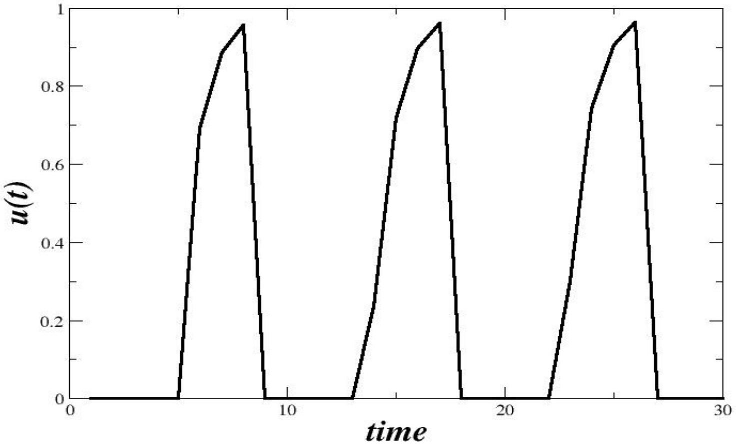
u(t)=membrane potential

p_r=refractory period

μ= leaky integrator constant



$p_r = 0$

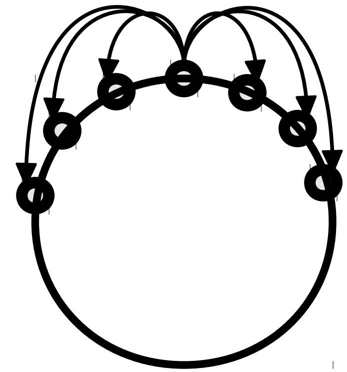


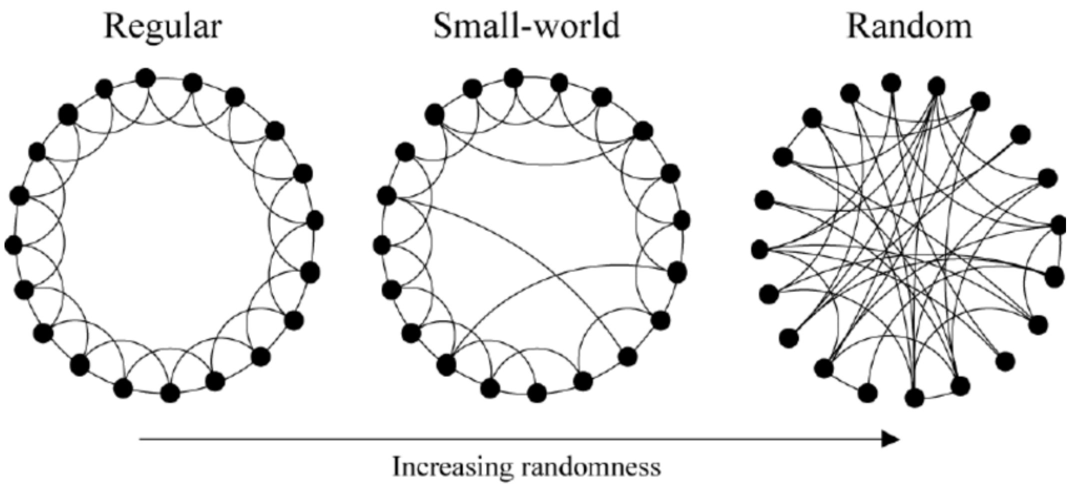
$p_r \neq 0$

2.2 Coupled LIF oscillators

$$\frac{du_i(t)}{dt} = \mu - u_i(t) + \frac{1}{R}\sum_{j=connect.} \sigma_{ij}\left[u_j(t) - u_i(t)\right]$$

$$u_i(t) \to 0, \quad when \quad u_i(t) > u_{th}$$





σ_{ij} = coupling strength, μ =1, u_{th}=0.98, N=1000
*Periodic boundary conditions on a ring
*Variables: σ, p_r, geometry

Olmi, Politi & Torcini, EPL, vol. 92, 60007 (2010)
Luccioli & Politi, PRL, vol. 105, 158104 (2010)

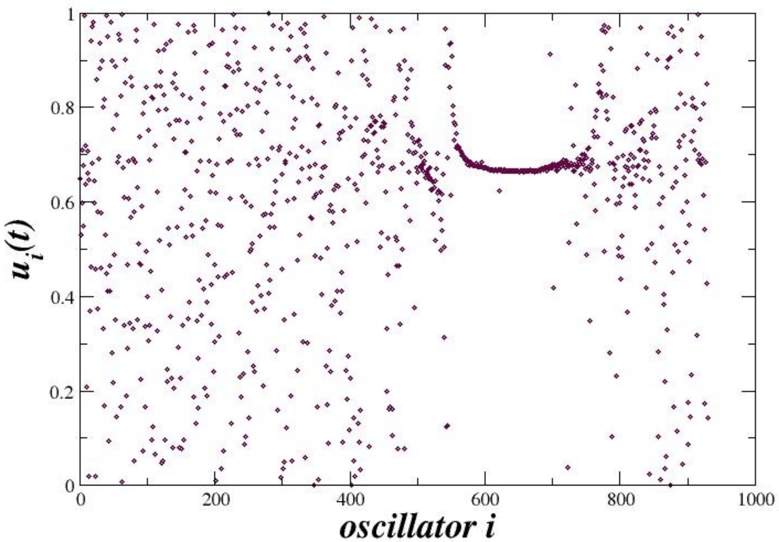
2.3 Coupled LIF Oscillators in 1D (ring)

$$\frac{du_i(t)}{dt} = \mu - u_i(t) + \frac{\sigma}{2R} \sum_{j=i-R}^{i+R} [u_i(t) - u_j(t)]$$

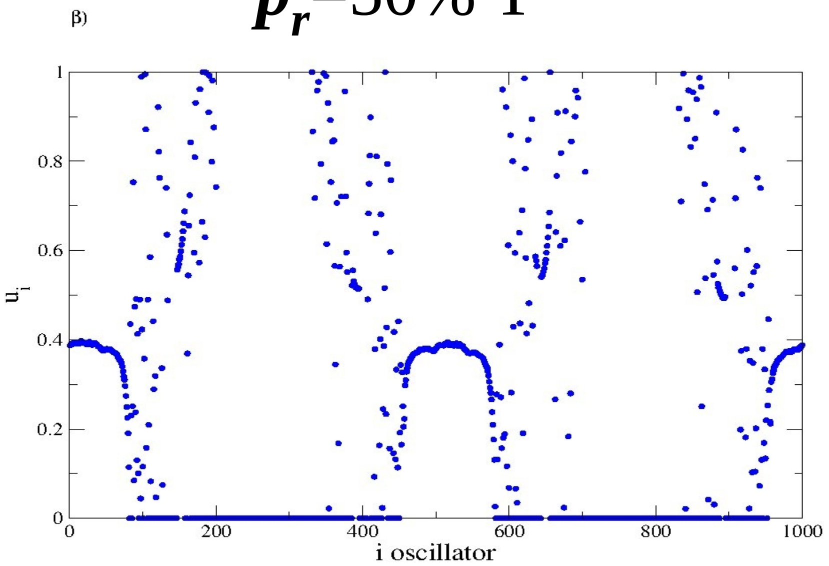
$$u_i(t) \to 0, \quad when \quad u_i(t) > u_{th}$$

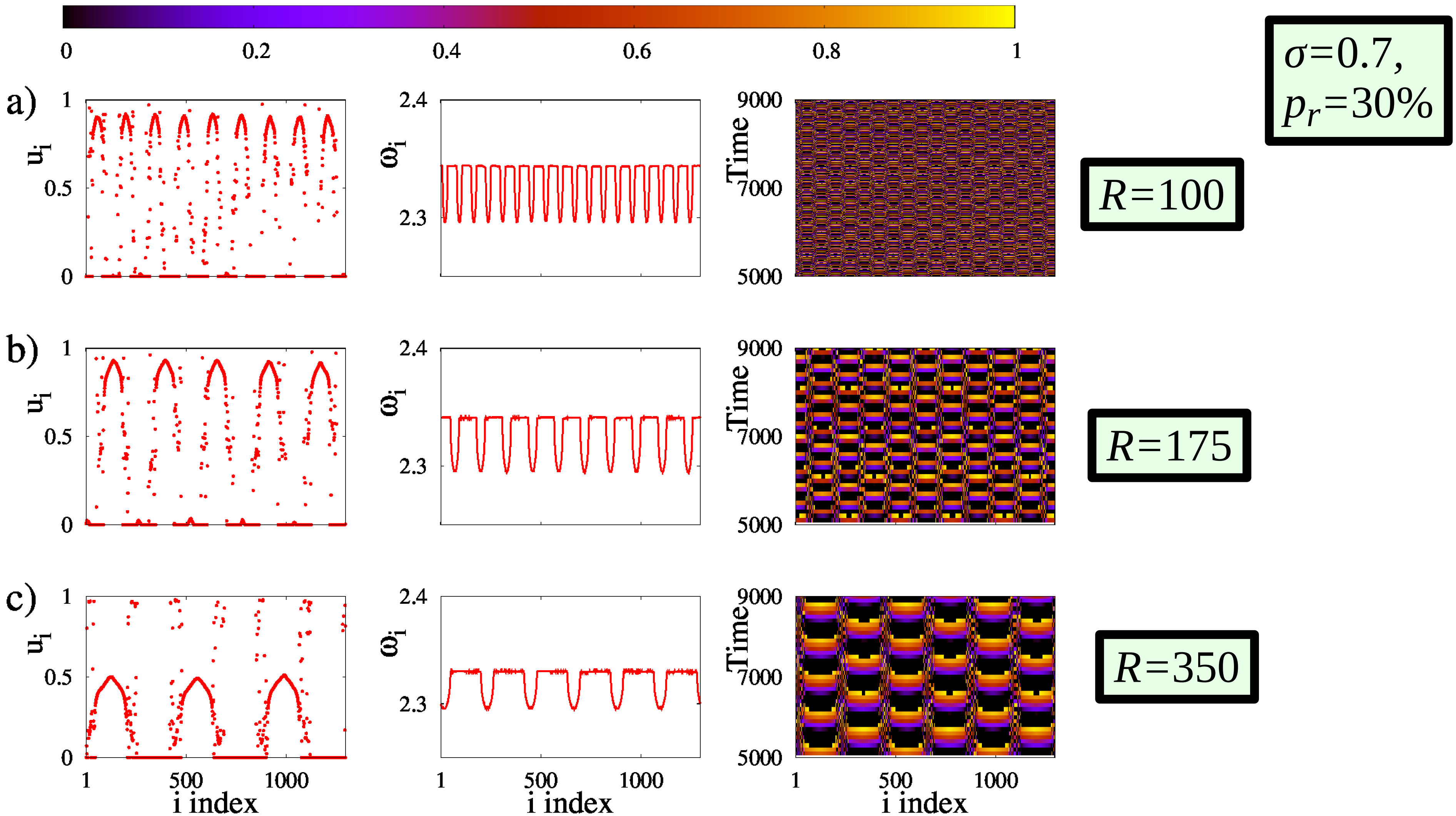
$\sigma = 0.656$
$R = 350$

a) Without refractory period
=> single chimera
$p_r = 0$



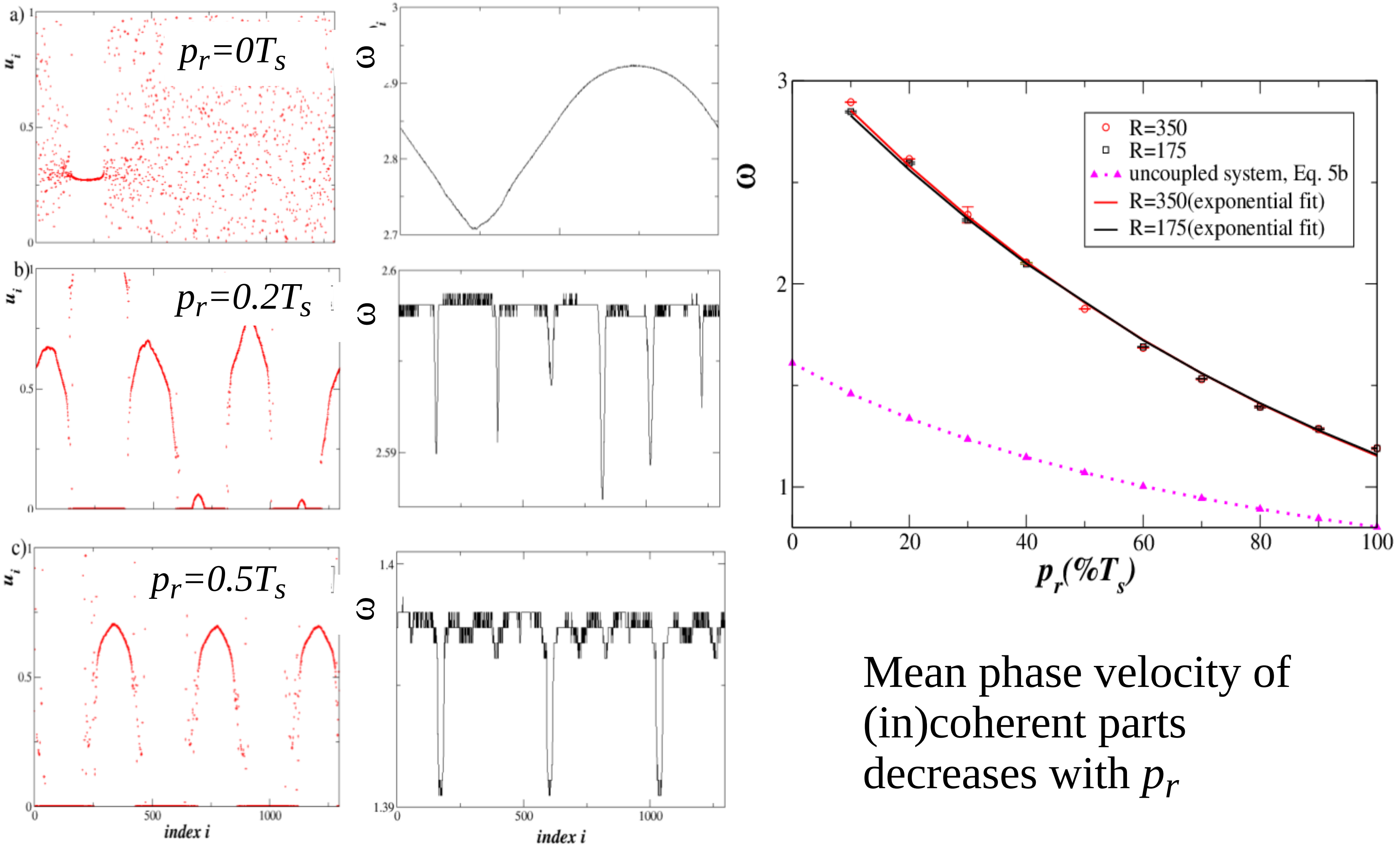
b) With refractory period
=> multi-chimera
$p_r = 50\%$ T





As $R\uparrow$ the number of (in)coherent parts decreases: Expected...

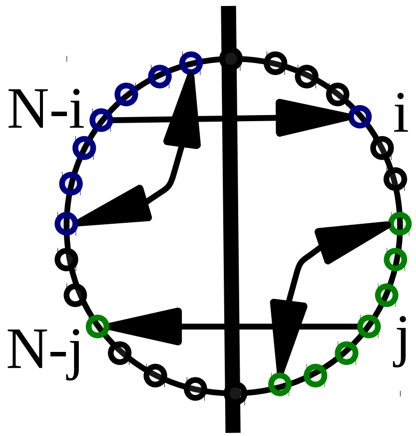
Parameter range for chimeras : $\sigma \in (0.5\ ,\ 0.8)$, $p_r \in (0T_s\ ,\ 1.0T_s)$



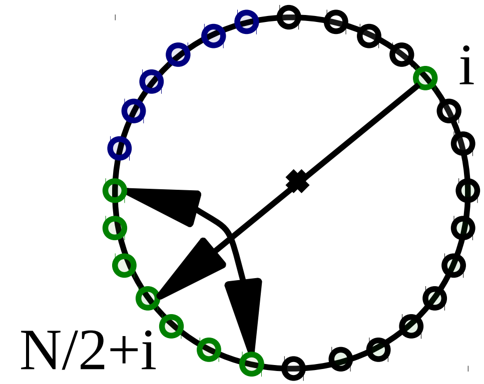
Mean phase velocity of (in)coherent parts decreases with p_r

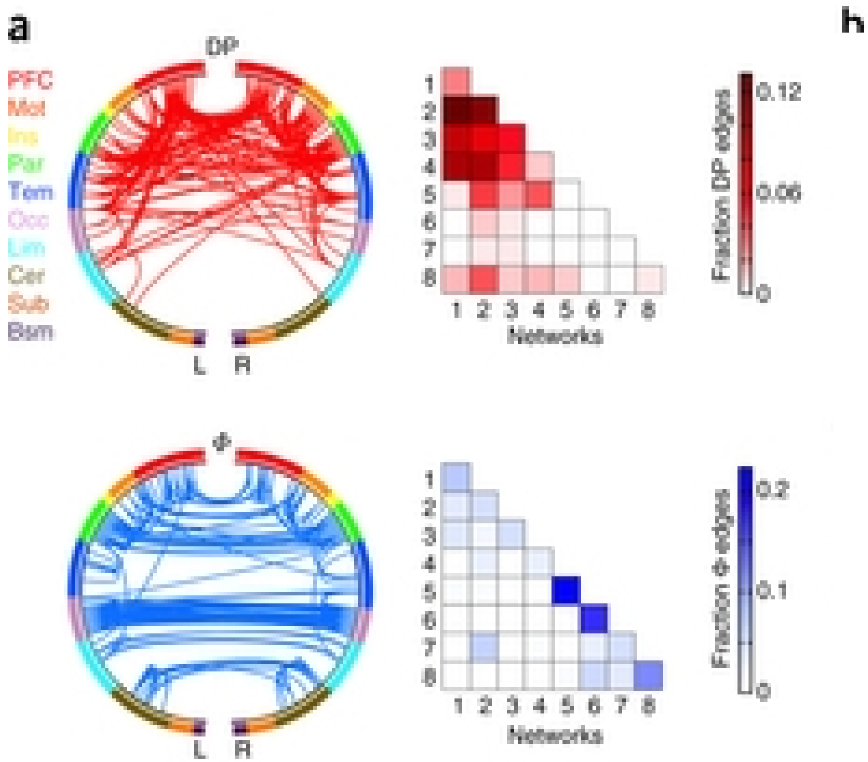
R=350, σ=0.7

2.4 Coupled LIF oscillators in various connectivity schemes



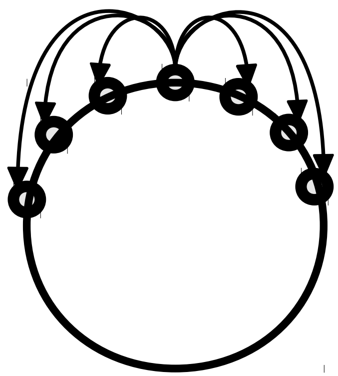
1D
Reflecting
Coupling
and
Diagonal
Coupling





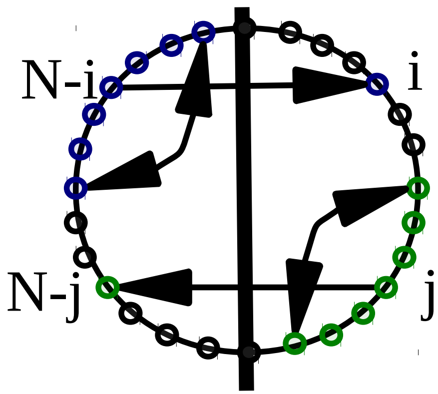
Finn et al.,
Nature Neuroscience,
Vol. 18, p. 1664 (2015)

2.4 Coupled LIF oscillators in various connectivity schemes



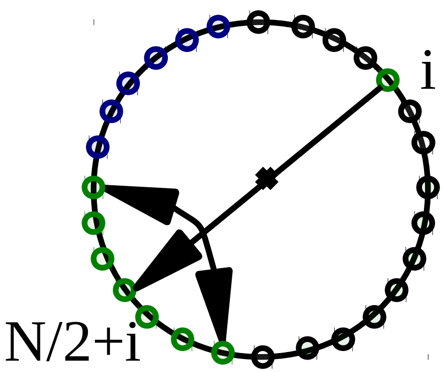
Non-local connectivity

$$\sigma_{ij} = \begin{cases} \sigma & \text{if } N-i-R \le j \le N-i-R \\ 0 & \text{otherwise} \end{cases}$$



Reflecting connectivity

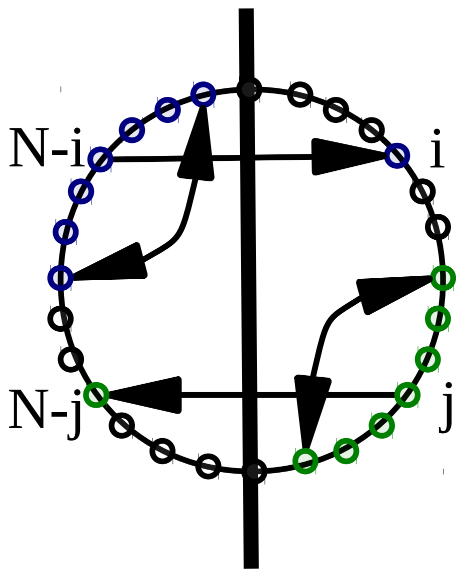
$$\sigma_{ij} = \begin{cases} \sigma & \text{if } N-i-R \le j \le N-i-R \\ 0 & \text{otherwise} \end{cases}$$

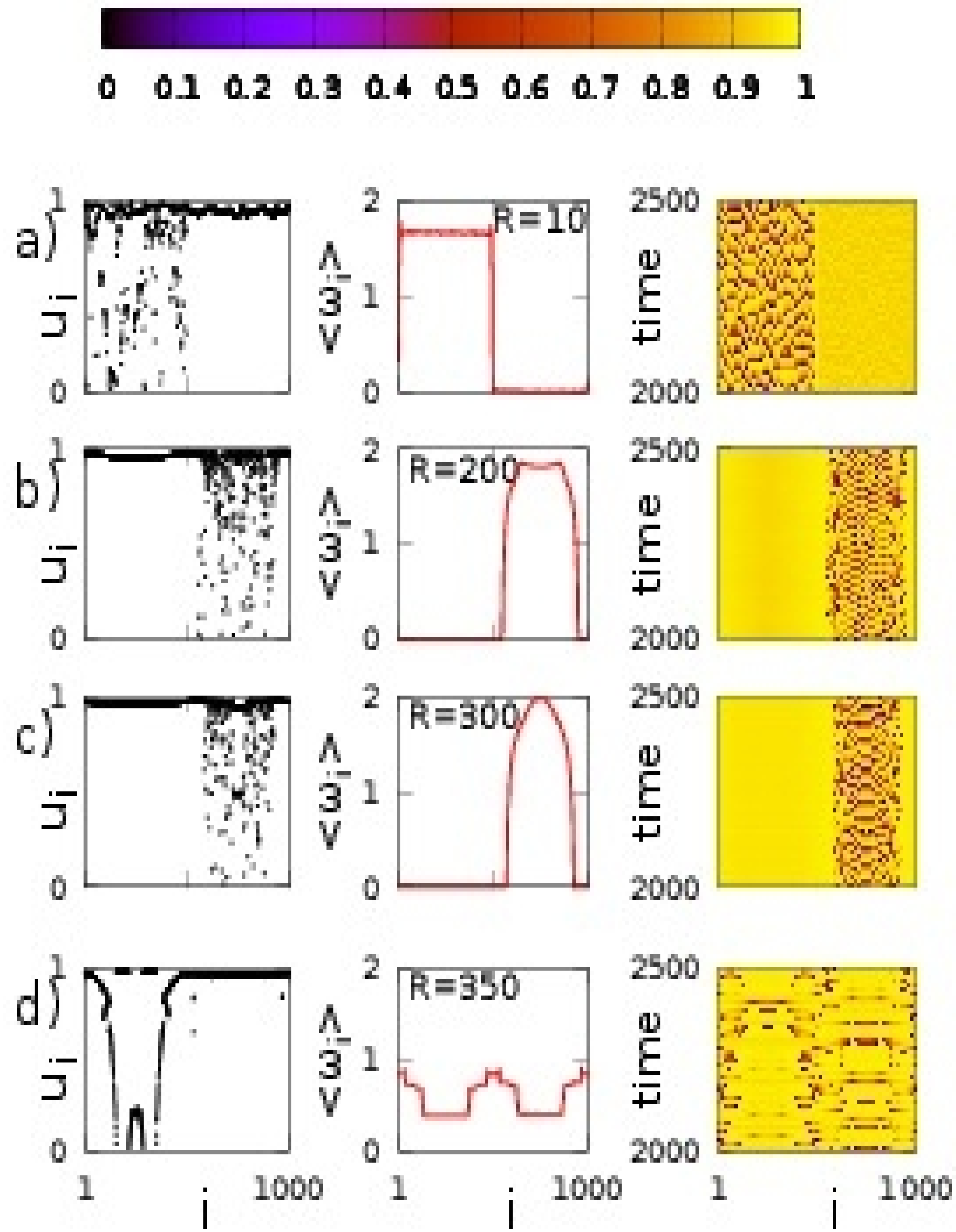


Diagonal connectivity

$$\sigma_{ij} = \begin{cases} \sigma & \text{if } \frac{N}{2}+i-R \le j \le \frac{N}{2}+i-R \\ 0 & \text{otherwise} \end{cases}$$

2.5 Reflecting Connectivity

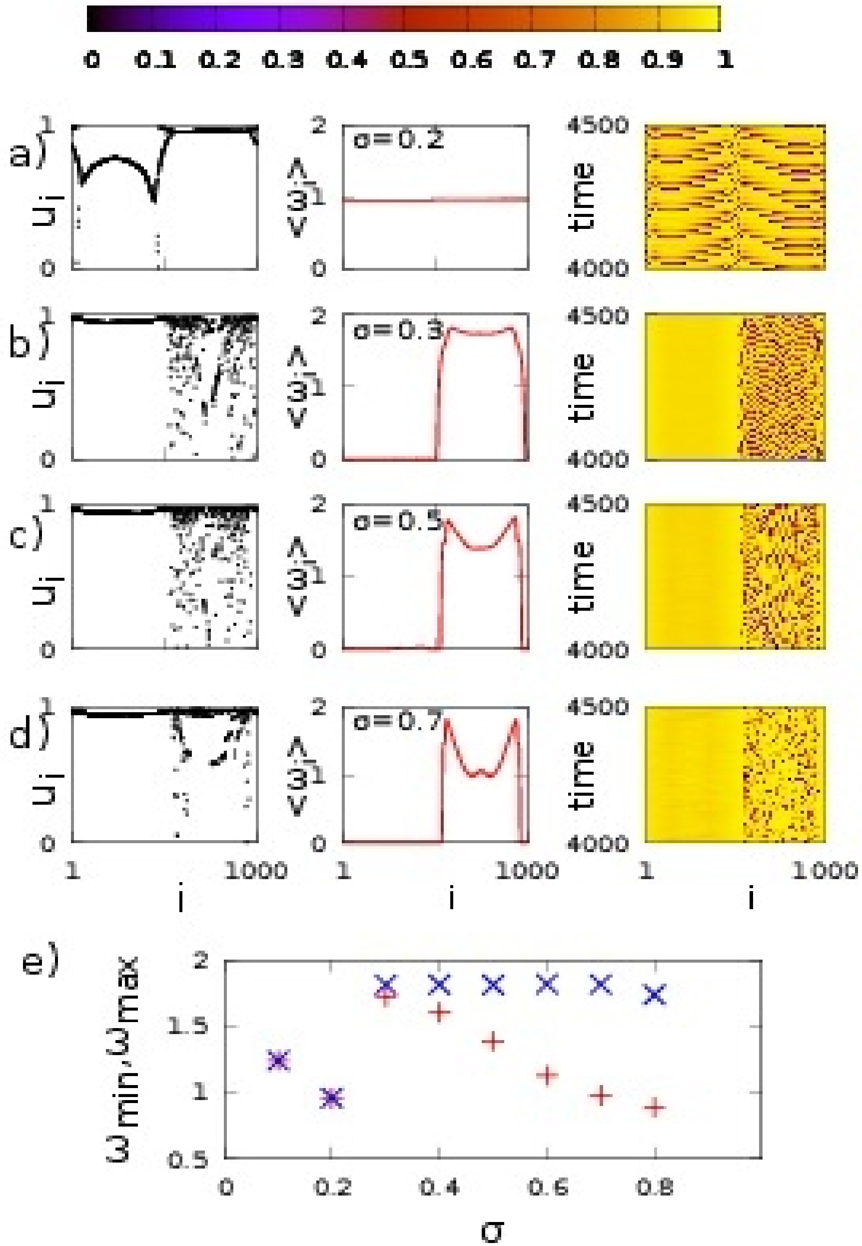




Confinement Phenomena: The activity gets confined in one semi-ring for small values of *R*. In the other semi-ring the elements stay near-threshold.

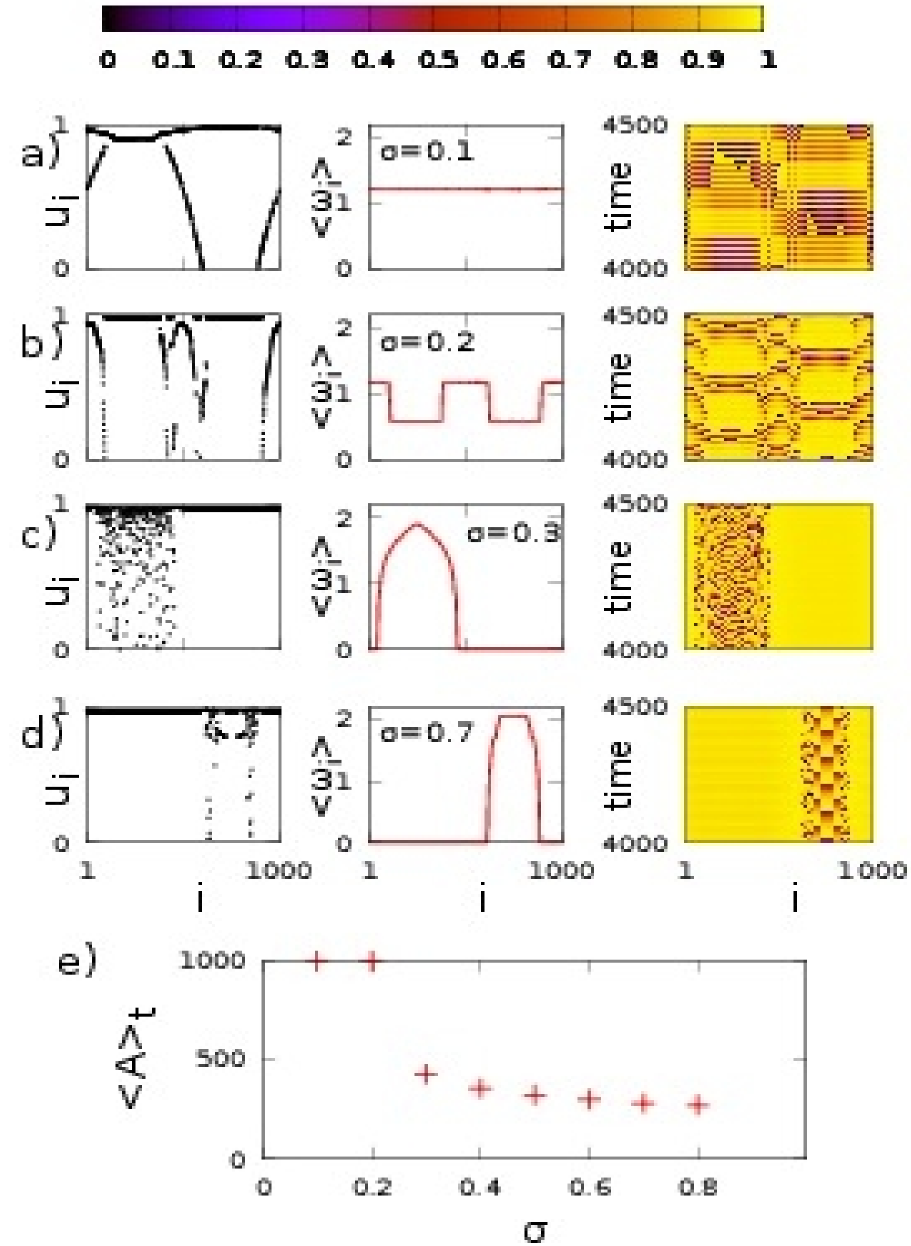
When $R \rightarrow N$ the activity extends to the entire system.

(σ=0.4, p_r=0, N=1000, μ=1.0)



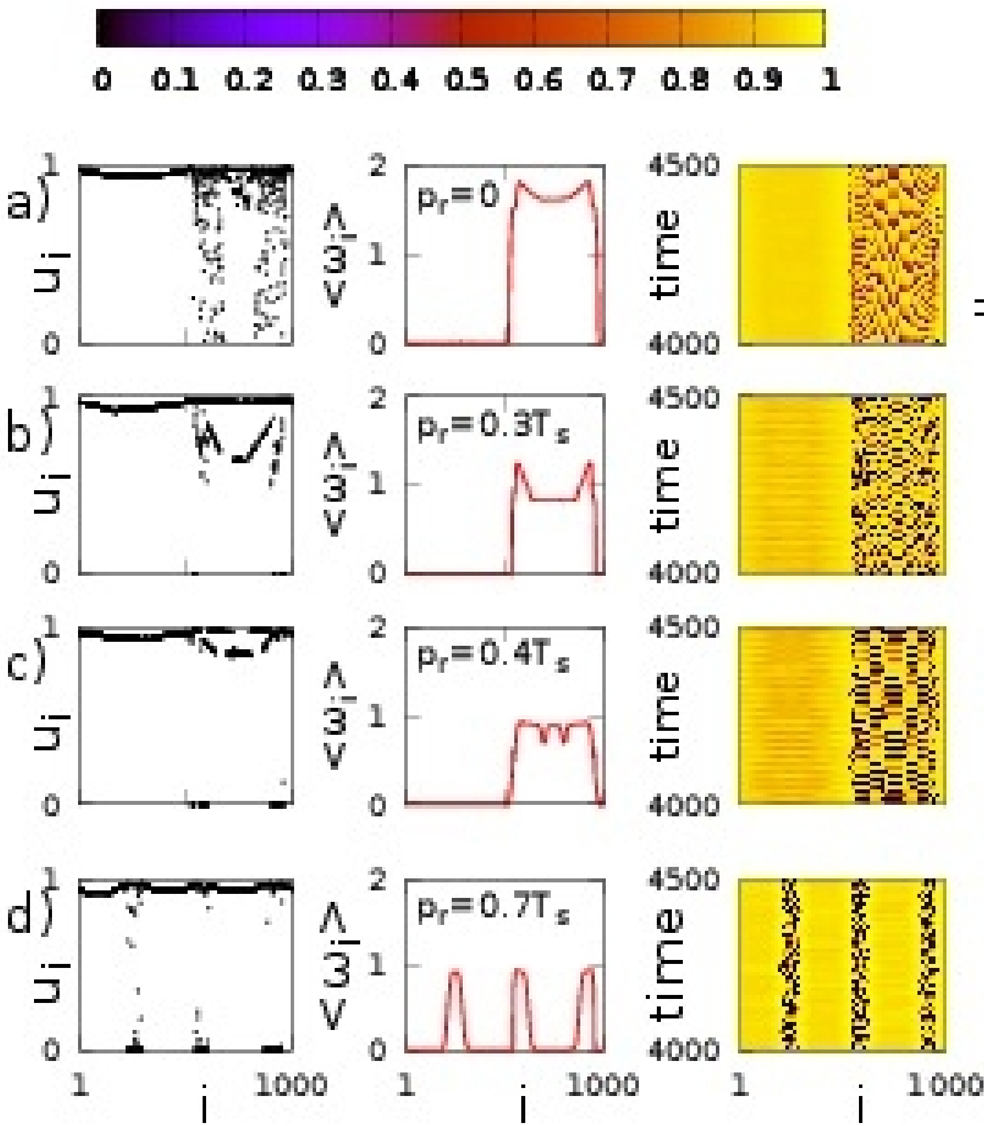
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
a)
b)
c)
d)
e)
σ=0.2
σ=0.3
σ=0.5
σ=0.7
u_i
<ω_i>
time
i
ω_min, ω_max
σ

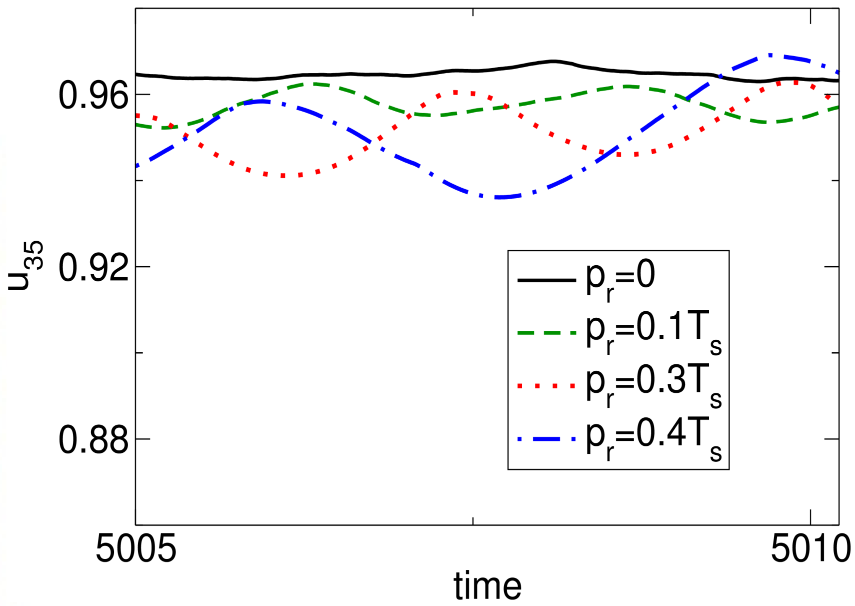
(*R=100*)



0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
a)
b)
c)
d)
e)
σ=0.1
σ=0.2
σ=0.3
σ=0.7
u_i
<ω_i>
time
i
<A>_t
σ

(R=300)

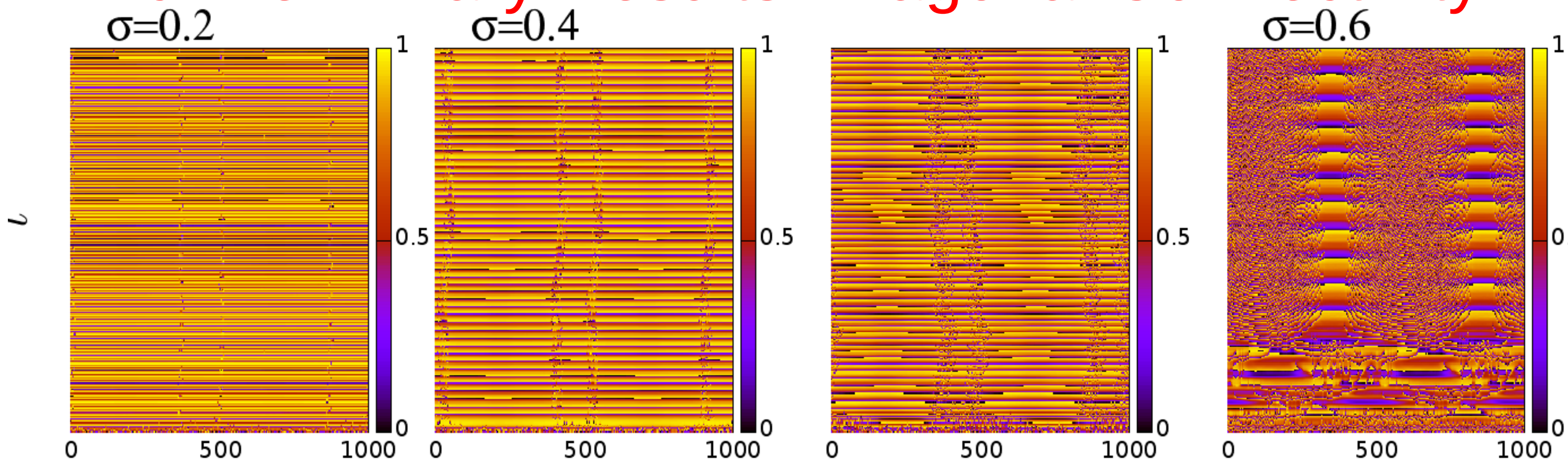




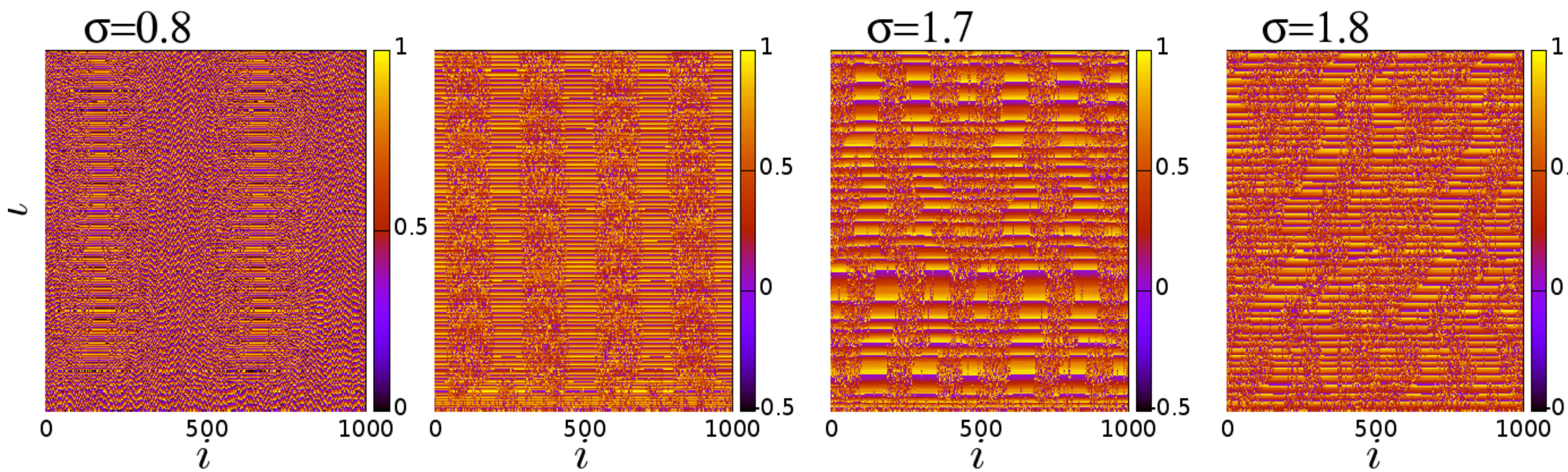
The near-threshold elements are not totally immobile, they perform short oscillations but stay near the threshold.

$\sigma=\mathbf{0.4}$, $R=100$, $N=1000$, $\mu=1.0$ *and* $u_{th}=0.98$

2.6 Preliminary Results: Diagonal Connectivity



few rebels → mobilization → mobilization+ → regime-hybride →

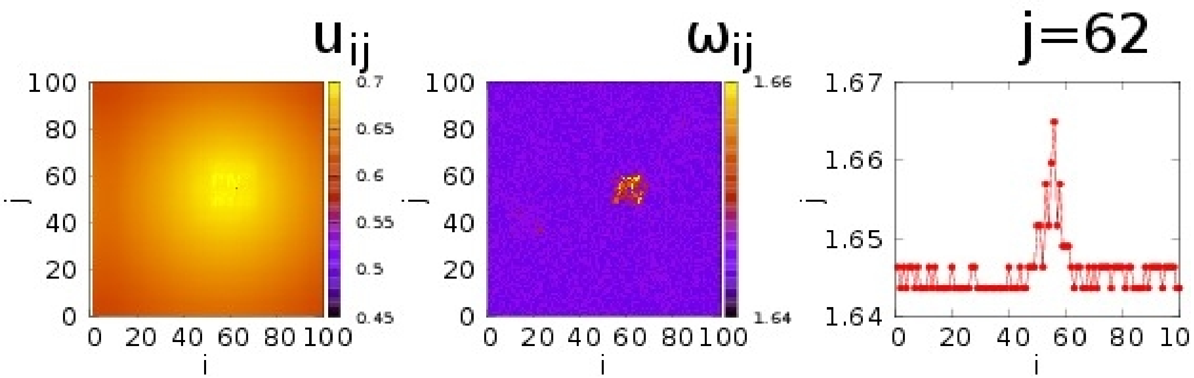


order suppression → chaos → regime2 → destabilization of regime2 → further dest.

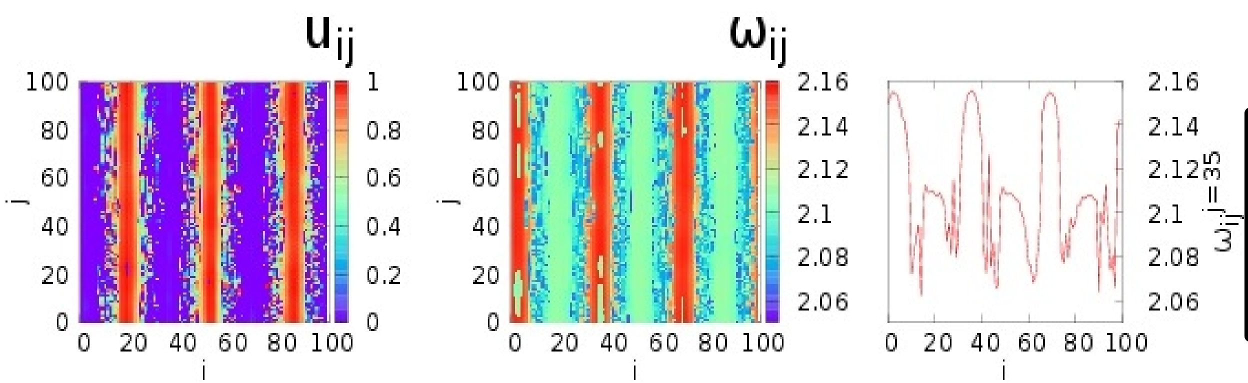
N=1000 oscillators, R=150+150+150+150, μ=1.0, uth=0.98

Small coupling σ => small incoherent regions; σ+ => larger incoherent regions; σ++ => multiplicity changes; σ++ => domains unstable, mixing.

2.7 Nontrivial generalizations in 2D & 3D



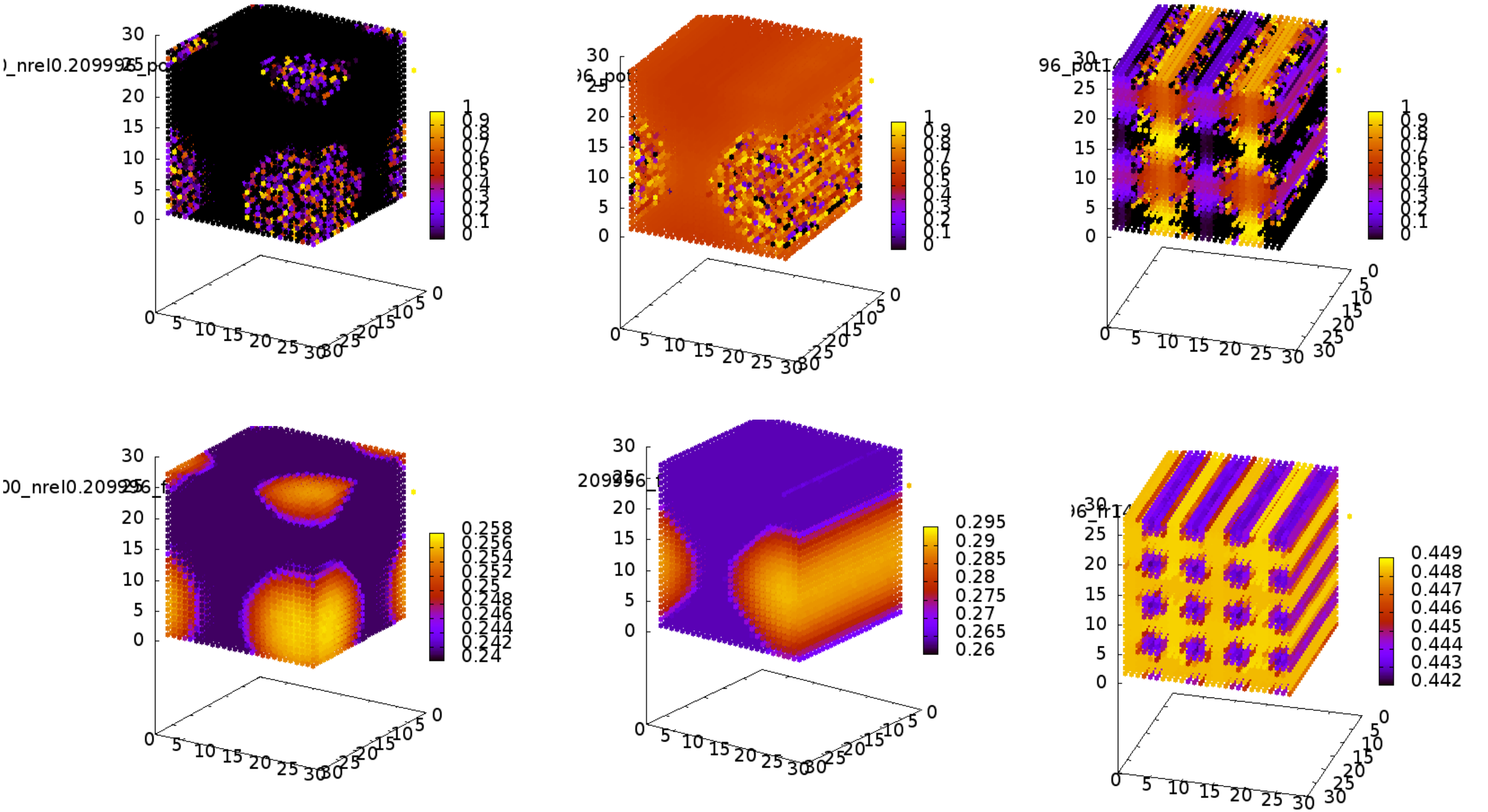
Direct generalization of 1D
(σ=0.1, R=10, p_r=0T_s)



Generalization of 1D but:
+ 2 coherent classes!
(σ=0.6, R=20, p_r=0.6T_s)

System size: N=100 X100, μ=1.0

2.8 Preliminary Results: 3D



Incoherent spot	Incoherent cylinder	Grid
(p_r=0.47, σ=0.1)	(p_r=0.47, σ=0.2)	(p_r=0.61, σ=0.7)

Size: 27 X 27 X 27 = 20000; T=0.21Ts; top=potential, bottom=mean phase velocity

(Y. Maistrenko et al., New Journal of Physics (2015): 3D-chimeras in Kuramoto model)

3.1 The FitzHugh Nagumo Model (1961):

[originates from the Hodgkin–Huxley model and models propagation of electrical signals in neurons]

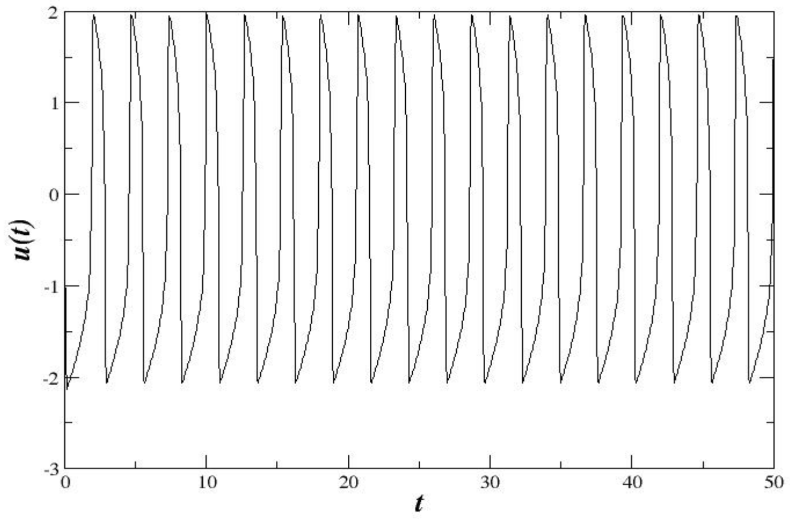
$$\epsilon \frac{du(t)}{dt} = u(t) - \frac{u^3(t)}{3} - v(t) + I(t)$$

$$\frac{dv(t)}{dt} = u(t) + \alpha$$

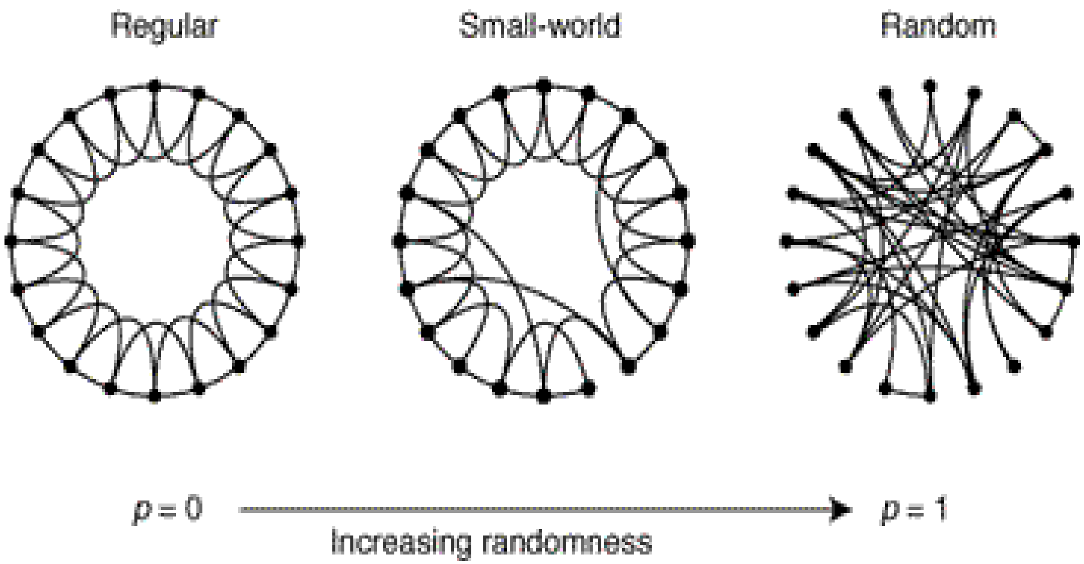
$\alpha = 0.5$
$\epsilon = 0.05$

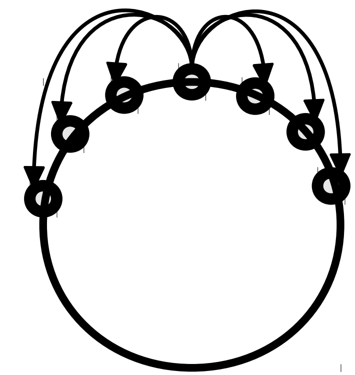
$I(t)$=const=0.5

$u(t)$=membrane potential (activator)
$v(t)$= recovery potential (inhibitor),
$I(t)$=external stimulus



3.2 Coupled FitzHugh Nagumo Oscillators (in a ring)



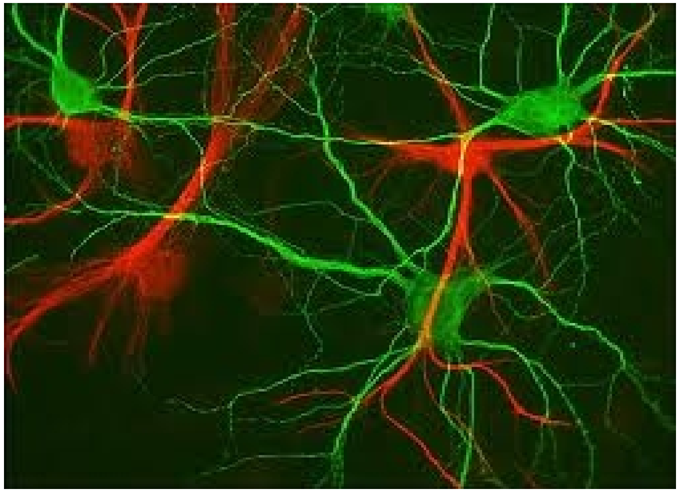


* With the current development on networks, a first approach is to put the oscillators in a ring

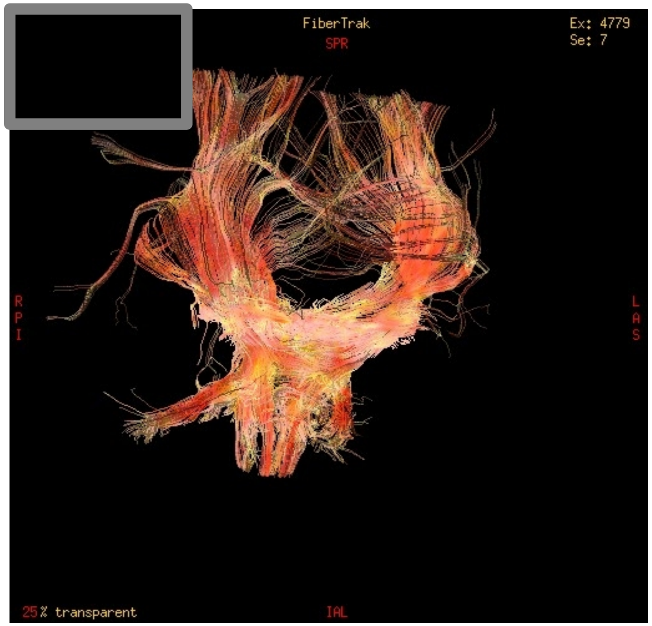
$$\epsilon \frac{du_i(t)}{dt} = u_i(t) - \frac{u_i^3(t)}{3} - v_i(t) + \frac{\sigma}{2R} \sum_{j=i-R}^{i+R} [u_j(t) - u_i(t)]$$

$$\frac{dv_i(t)}{dt} = u_i(t) + \alpha + \frac{\sigma}{2R} \sum_{j=i-R}^{i+R} [v_j(t) - v_i(t)]$$

[Parenthesis on Brain Connectivity:



Neurons: are electrically excitable cells which process and transmit information through electrical signals
- ***soma*** (contains the nucleus, typical 25µm)
- ***dendrites*** (receive signals)
- ***axons*** (connect neurons and transmit signals, size 1µm, max 1m!)
- ***axon terminals*** (contain synapses to communicate the signal)

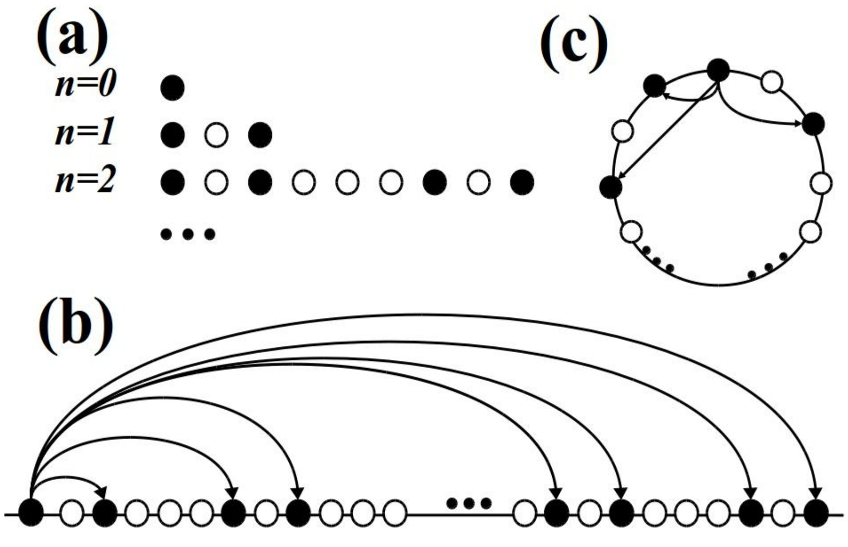


DTI – MRI: Neuron axons in 3D representation

- Resolution: 1-3mm
- Fractal dimensions of the neuron axons network: 2.5-2.6
- Different correlations and fractality for neurodegenerative disorders

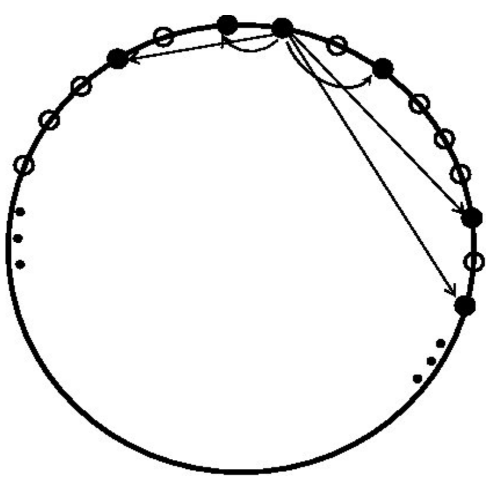
]

Coupling on Fractal Networks

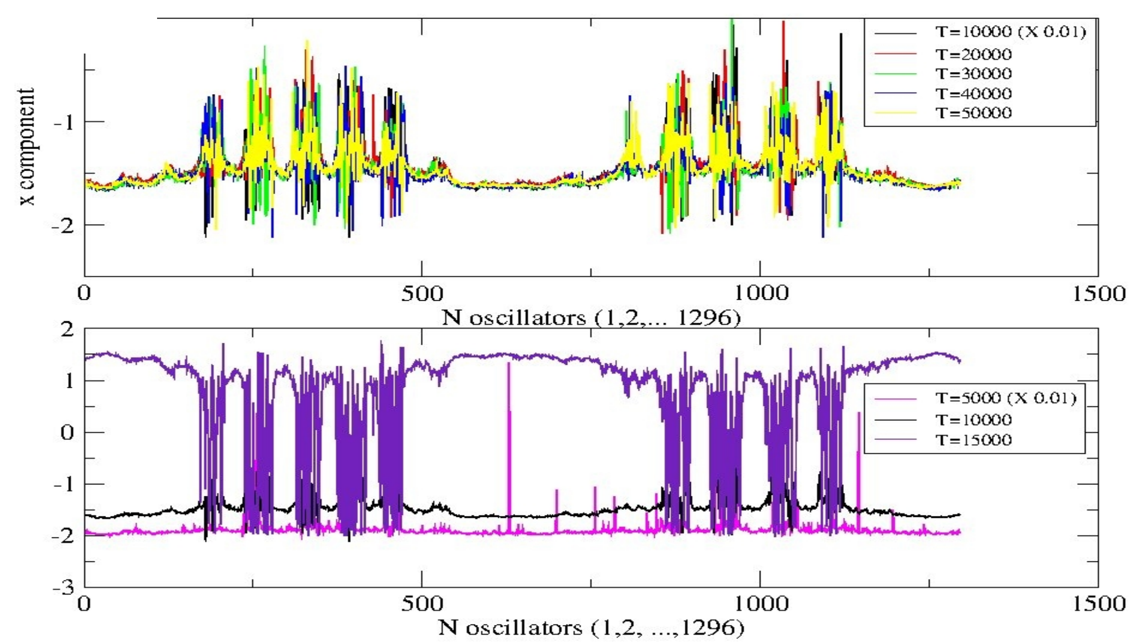


See: movie-fhn-fractal

Nested Chimera States



Random Fractal (2) connectivity ln4/ln6



Appearance and destruction of a nested/ramified/hierarchical chimera state

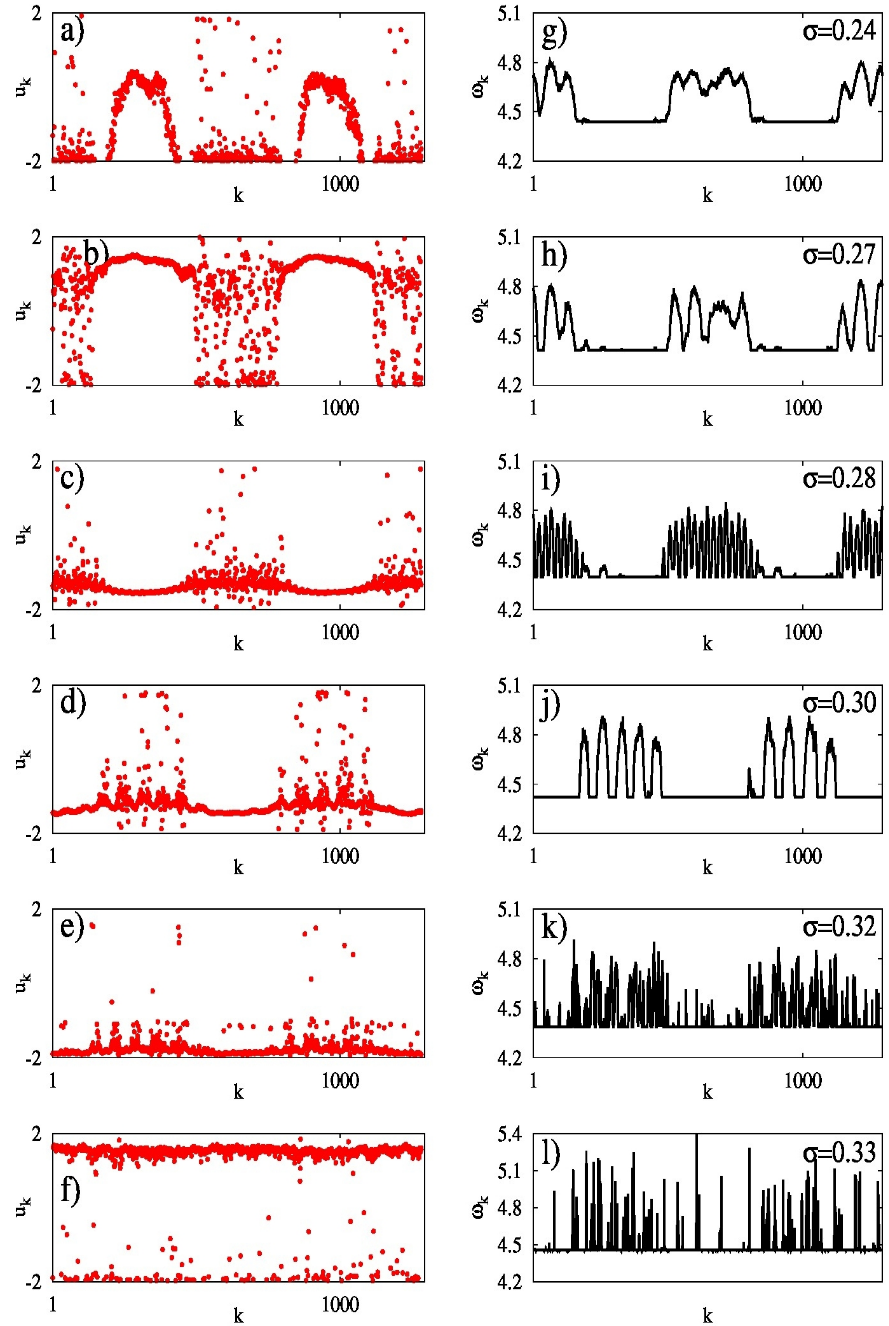
Ramifications are due to fractal connectivity

σ= coupling strength

For σ>> we drive to synchronization

Omelchenko et al. PRE 2015

See movie: chimera-fractal



The role of spatial correlations in connectivity

I. Non-local connectivity
II. Asymmetric nonlocal
III. Fractal-hierarchical connectivity
IV. Reflecting connectivity
V. Diagonal connectivity
VI. Modular networks connectivity

→ Spatial correlations in connectivity

1. Random connectivity networks
2. Random values of the coupling strengths
3. Small world networks
…
4. Other realistic networks

→ If noise is added in the connectivity, chimera state starts disintegrating

4. Conclusions

- Chimera States in FHN and LIF neuron dynamics
- Spiking regime induces chimera states
- Nonlocal (spatially correlated) connectivity produces chimera states
- Hierarchical connectivity: traveling chimeras

Open Problems

- Connection of synchronization patterns with memory and cognition
- Interplay between topology and dynamics
- Spatial correlations in the connectivity => chimera states???

- Time dependent connectivity
- Apoptosis of neurons
- Influence of external forces on chimera states
- Influence of initial conditions...

Collaborations & Thanks

NCSR Demokritos

* Panayotis Katsaloulis
* Dimitris Verganelakis
* Theodore Kasimatis
* Nefeli Tsigkri-DeSmedt
* Johanne Hizanidis
* Astero Provata

TU Berlin

* Alexander Smith
* Thomas Isele
* Iryna Omelchenko
* Philipp Hoevel
* Eckehard Schoell

THANK YOU FOR YOUR ATTENTION !

Selected Recent Publications:

-- P. A., Katsaloulis P and Verganelakis DA, "Dynamics of chaotic maps for modelling the multifractal spectrum of human brain DTI Images", Chaos Solitons & Fractals 45 , 174, 2012.

-- Katsaloulis P, Hizanidis J, Verganelakis DA and P. A., "Complexity Measures and Noise Effects on DTI-MRI images in the human brain", Fluctuations & Noise Letts. 11, 1250032, 2012.

--Katsaloulis P, Ghosh A, Philippe AC, P. A. and Deriche R,"Fractality in the neuron axonal topography of the human brain based on 3-D diffusion MRI", EPJB 85, 150, 2012.

--Omelchenko I, P. A., Hizanidis J, Schöll E and Hövel P, "Robustness of chimera states for coupled FitzHugh-Nagumo Systems" PRE 91, 022917, 2015.

– Hizanidis J, Panagakou E, Omelchenko I, Schöll E, Hövel P and P. A., "Chimera States in Population Dynamics Networks" PRE 92, 012915, 2015.

– Isele T, Hizanidis J, P. A. and Hövel P, "Controlling Chimera States: The Influence of Excitable Units " PRE 93, 022217, 2016.

- Tsigkri-DeSmedt, Hizanidis J, Hövel P and P. A., "Multichimera Chimera States in the LIF model with nonlocal and hierarchical connectivity" EPJST 225, 1149-1164, 2016.

Motivating Questions:

Theory:

- Why chimera numerical evidence is mostly linked with neuron-related models?

- Spiking dynamics essential in neuron models: Is it also essential for the production of chimera states?

- Role of connectivity and the formation of chimera states ?
  Are spatial correlations important for the formation of chimera states??

Applications:

- Are chimera states, as patterns formed under certain (external) conditions in co-operation with internal dynamics+connectivity, relevant in memory & cognition-related activities.
- Is the form of chimera patterns relevant in brain neurological/ neurodegenerative disorders?
- Can it be revealed in experiments of brain partial activity (such simple task Experiments: parroting, eye movement, finger tapping)?

(Synchronization patterns ?? Brain Activity)

Specific stable synchronisation patterns are formed under specific “connectivity, coupling, initial conditions, external stimulii, etc”. (Synchronization patterns ? Memory? Cognition)

Under the same external stimuli the same synchronization patterns reappear if the connectivity scheme and couplings are unchanged . (Memory?)

If connectivity changes slightly, the pattern remains slightly changed (Fainting Memory ?).

If connectivity changes a lot the synchronization patterns are destroyed (Memory Loss ?)

When synchronization pattern appears chemistry is recalled...